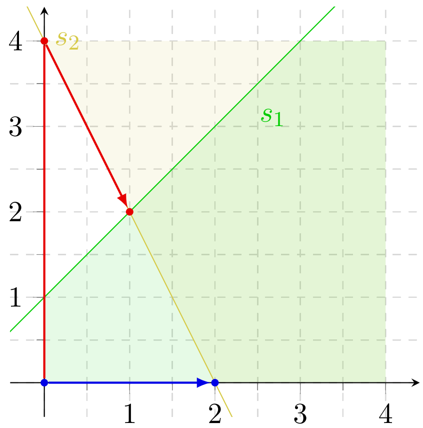
1 The constraints can be visualized as follows:



$$\begin{aligned} -x + y &= s_1 & s_1 &\leq 1 \\ -2x - y &= s_2 & s_2 &\leq -4 \end{aligned}$$

a) As a variable order, the following calculation uses $s_1 > s_2 > x > y$. At the start all variables are assigned 0, and the following initial tableau is constructed:

$$\begin{array}{c} \\ s_1 \\ s_2 \end{array} \begin{array}{c} \begin{array}{cc} x & y \end{array} \\ \begin{pmatrix} -1 & 1 \\ -2 & -1 \end{pmatrix} \end{array}$$

(1) The bound for s_2 is violated. In order to decrease s_2, we need to increase x or y. Both are suitable for pivoting, because they do not have bounds. We pick y (to stick to Bland's Rule). This results in the following new tableau:

$$\begin{array}{c} \\ s_1 \\ y \end{array} \begin{array}{c} \begin{array}{cc} x & s_2 \end{array} \\ \begin{pmatrix} -3 & -1 \\ -2 & -1 \end{pmatrix} \end{array}$$

Now s_2 is assigned to its bound -4, and x is kept at 0. According to the new tableau, we calculate theremaining values for the new assignment $s_1 = 4$, $y = 4$.

(2) Unfortunately now the bound for s_1 is violated. In order to decrease s_1, we have to increase x or s_2. Since s_2 is already at its upper bound, only x is suitable. We obtain the following tableau:

$$\begin{array}{c} \\ x \\ y \end{array} \begin{array}{c} \begin{array}{cc} s_1 & s_2 \end{array} \\ \begin{pmatrix} -\frac{1}{3} & -\frac{1}{3} \\ \frac{2}{3} & -\frac{1}{3} \end{pmatrix} \end{array}$$

We set s_1 to its bound 1, while s_2 remains at -4. This implies $x = 1$ and $y = 2$. With this assignment the bounds are satisfied. In the picture, the assignments obtained in this Simplex search are shown in red.

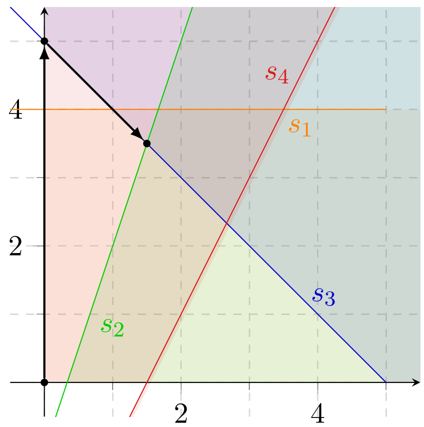
b) Suppose we instead use a variable order where $y > x$, like $y > x > s_2 > s_1$.

(1) The order does not change the initial tableau; again s_2 violates its bounds and both x and y are suitable. But according to Bland's Rule we now pivot with x, to obtain

$$\begin{array}{c} \\ s_1 \\ x \end{array} \begin{array}{c} \begin{array}{cc} s_2 & y \end{array} \\ \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} \end{pmatrix} \end{array}$$

We set $s_2 = -4$ and leave $y = 0$, which implies $s_1 = -2$ and $x = 2$, which satisfies the bounds. In the picture, this Simplex search are shown in blue.

2 The constraints can be visualized as follows:



$$\begin{array}{rcr} y & \leq & 4 \\ -3x + y & \leq & -1 \\ -x - y & \leq & -5 \\ 2x - y & \leq & 3 \end{array}$$

As a variable order, the following calculation uses $s_1 > \cdots > s_4 > x > y$. At the start all variables are assigned 0, and the following initial tableau is constructed:

$$\begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \\ s_4 \end{array} \begin{array}{c} \begin{array}{cc} x & y \end{array} \\ \begin{pmatrix} 0 & 1 \\ -3 & 1 \\ -1 & -1 \\ 2 & -1 \end{pmatrix} \end{array} \begin{array}{l} \\ s_1 \leq 4 \\ s_2 \leq -1 \\ s_3 \leq -5 \\ s_4 \leq 3 \end{array}$$

(1) The bounds for s_2 and s_3 are violated. Choosing s_3, both x and y are suitable for pivoting. We pick y (to stick to Bland's Rule). This results in the following new tableau:

$$\begin{array}{c} \\ s_1 \\ s_2 \\ y \\ s_4 \end{array} \begin{array}{c} \begin{array}{cc} x & s_3 \end{array} \\ \begin{pmatrix} -1 & -1 \\ -4 & -1 \\ -1 & -1 \\ 3 & 1 \end{pmatrix} \end{array} \begin{array}{l} \\ s_1 \leq 4 \\ s_2 \leq -1 \\ s_3 \leq -5 \\ s_4 \leq 3 \end{array}$$

Now s_3 is assigned to its bound -5, and x is kept at 0. The values of the remaining variables are computed from the new tableau: $s_1 = 15$, $s_2 = 5$, $y = 5$, $s_4 = -5$.

(2) The new assignment violates the bounds for s_1 and s_2. When choosing s_2 (again according to Bland's Rule), only x is suitable for pivoting. (In order to decrease s_2 we would have to *in*crease s_3, but s_3 is already at its upper bound and thus not suitable.) This results in the following new tableau:

$$\begin{array}{c} \\ s_1 \\ x \\ y \\ s_4 \end{array} \begin{array}{c} \begin{array}{cc} s_2 & s_3 \end{array} \\ \begin{pmatrix} \frac{1}{4} & -\frac{3}{4} \\ -\frac{1}{4} & -\frac{1}{4} \\ \frac{1}{4} & -\frac{3}{4} \\ -\frac{3}{4} & \frac{1}{4} \end{pmatrix} \end{array} \begin{array}{l} \\ s_1 \leq 4 \\ s_2 \leq -1 \\ s_3 \leq -5 \\ s_4 \leq 3 \end{array}$$

Now s_2 is assigned to its bound -1, and s_3 remains at -5. The values of the remaining variables are computed from the new tableau: $s_1 = 3\frac{1}{2}$, $x = 1\frac{1}{2}$, $y = 3\frac{1}{2}$, $s_4 = -\frac{1}{2}$.

This assignment satisfies all bounds. The solution search is illustrated in the picture.

3 See the file `tsp.py`.

4 Postponed to next week, stay tuned!