

[3] 1 Draw the equality graphs for the following equality logic conjunctions, identify all simple cycles, and decide whether the formulas are satisfiable.

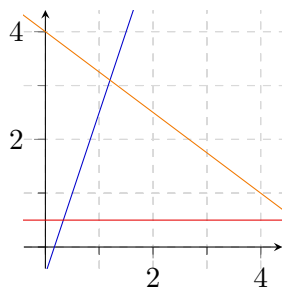
(a) $x_1 = x_3 \wedge x_1 \neq x_4 \wedge x_3 \neq x_7 \wedge x_4 \neq x_6 \wedge x_6 \neq x_7 \wedge x_5 = x_9 \wedge$
 $x_5 = x_7 \wedge x_8 \neq x_9 \wedge x_9 = x_{10} \wedge x_7 = x_9 \wedge x_5 \neq x_8$

(b) $x_1 = x_2 \wedge x_2 = x_3 \wedge x_3 = x_4 \wedge x_4 \neq x_6 \wedge x_1 \neq x_5 \wedge x_6 \neq x_1 \wedge x_4 = x_9 \wedge$
 $x_4 = x_1 \wedge x_1 \neq x_9 \wedge x_6 = x_7 \wedge x_7 = x_8 \wedge x_5 \neq x_8 \wedge x_2 = x_9 \wedge x_4 = x_8$

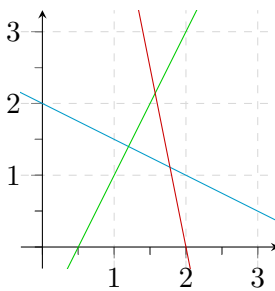
2 Consider the following six sets of linear arithmetic constraints:

<p>(1) $4 - x \leq 2y$ $y \leq 2x - 1$ $y \leq 10 - 5x$</p>	<p>(2) $4y \leq 11 - 4x$ $9 - 4x \leq 4y$ $1 - x \leq y$</p>	<p>(3) $2y \leq 4 - x$ $y \leq 2x - 1$ $y \leq 10 - 5x$</p>
--	---	--

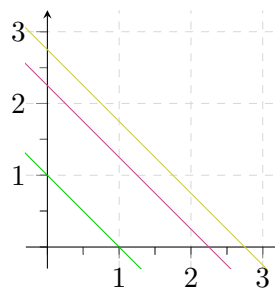
<p>(4) $4y \leq 11 - 4x$ $9 - 4x \leq 4y$ $y \leq 1 - x$</p>	<p>(5) $4y \leq 16 - 3x$ $1 \leq 2y$ $2y \leq 6x - 1$</p>	<p>(6) $1 \leq x$ $y \leq 2$ $x \leq y$</p>
---	--	--



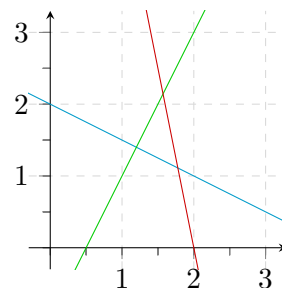
(A)



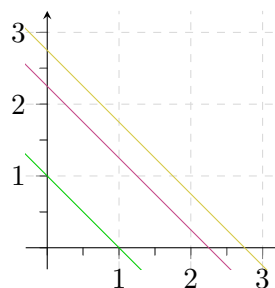
(B)



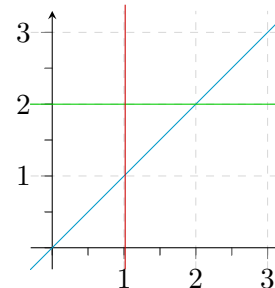
(C)



(D)



(E)



(F)

- [1] (a) For each constraint set **(1)**–**(6)** pick one of the diagrams **(A)**–**(F)** to visualize its solution space.
- [1] (b) Which of the problems are bounded?
- [2] (c) Determine from the drawings which problems are satisfiable over \mathbb{R}^2 and \mathbb{Z}^2 , and how many solutions there are over the respective domain.
- [3] 3 Apply the Branch and Bound approach to check satisfiability of the constraints **(5)** and **(6)** of the previous exercise over \mathbb{Z}^2 . You can use any solver over the reals, for instance your implementation of Simplex or Z3.
- [4] * 4 Implement a Simplex solver for a set of linear inequalities of the form $A\vec{x} \leq \vec{b}$.

(This might admittedly be tricky. Details of the DPLL(T) Simplex algorithm can be found in the following technical report: Bruno Dutertre and Leonardo de Moura: *Integrating Simplex with DPLL(T)*, Technical Report SRI-CSL-06-01, SRI International, 2006.)