



- [3] 1 Consider the following system of linear inequalities:

$$\begin{aligned}y &\leq x \\ 3 - x &\leq y\end{aligned}$$

Compute a bound for integer solutions as explained in the lecture.

- 2 Consider the following system of linear inequalities (cf. slide 14 of week 8):

$$\begin{aligned}-4x - 5y &\leq -10 \\ 2x - 5y &\leq 0 \\ -x + 5y &\leq 21 \\ -7x + y &\leq -4 \\ 5x + 2y &\leq 25\end{aligned}$$

- [2] (a) Use (an implementation of) the DPLL(T) Simplex algorithm (e.g. **this one**) to find a solution to the problem in \mathbb{R}^2 . Derive a Gomory cut from the solution, and draw it in the diagram.
- [2] (b) Run DPLL(T) Simplex on this modified problem, and repeat the procedure until you found a solution in \mathbb{Z}^2 .
- [3] 3 The schedule for the IT helpdesk staff of the Loanly Euro banking group is currently set up in a manual process using excel sheets; however, there are scenarios where the generated schedule violates some constraints. In order to deal with this issue, scheduling should be automated. The constraints are as follows:
- (a) There are three types of 8-hour shifts: morning shift (06:00–14:00), afternoon shift (14:00–22:00), night shift (22:00–06:00).
 - (b) The staff consists of 7 persons who work morning, afternoon, and night in 8-hour shifts to provide a service that operates 24 hours a day and 7 days a week.
 - (c) The morning and afternoon shifts should include two employees.
 - (d) The night shift is done by one person.
 - (e) Each employee is required to work 40 hours per week.
 - (f) Employees must not work on a day shift after a night shift.
 - (g) Each employee must have at most one shift per day.
 - (h) Adam and Ben don't do night shifts.
 - (i) Dora does not work on weekends.
 - (j) Emma and Chris should not work together since they always fight.

Can you find a week schedule for the staff, using a linear integer arithmetic encoding?

- [3] ★ 4 Propose how a problem can be solved using SMT with linear arithmetic that was not covered in the lecture so far.

You do *not* have to solve it, but describe in detail how it could be done. In particular, describe all variables and constraints that are needed. (To exclude super easy problems let's say that the encoding should require at least 10 variables.)