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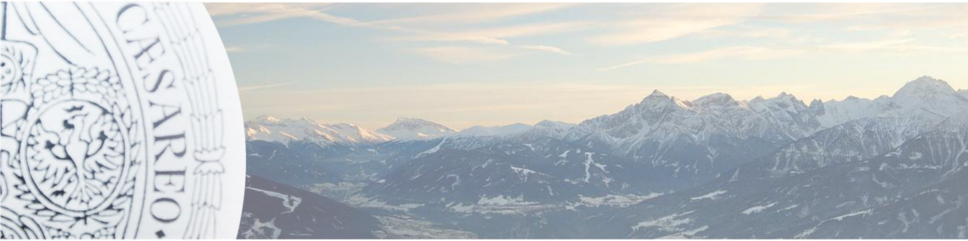

SAT and SMT Solving

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lecture 10
SS 2019

Outline

- Summary of Last Week
- Bit Vectors

Bounds for Satisfiability

Aim: Find Bound B

$$A\vec{x} \leqslant \vec{b} \text{ satisfiable} \iff (A\vec{x} \leqslant \vec{b} \quad \text{and} \quad \forall i.\ -B \leqslant \vec{x}_i \leqslant B) \text{ satisfiable}$$

Approach

1. represent $\{\vec{x} \mid A\vec{x} \leqslant \vec{b}\}$ as $hull(X) + cone(V)$
   - increase dimension by 1: represent $\{\vec{x} \mid A\vec{x} \leqslant \vec{b}\}$ by $\{\vec{z} \mid A\vec{z} \leqslant \vec{0}\}$
   - use FMW to represent $\{\vec{z} \mid A\vec{z} \leqslant \vec{0}\}$ as $cone(V)$
2. derive bound B for hull + cone representation

Theorem (Farkas, Minkowski, Weyl)

A cone C is polyhedral iff it is finitely generated

Corollary (for 2)

If $|c| \leqslant b$ for all coefficients c of vectors in $X \cup V$ then for $B := b \cdot (1 + n)$ have

$$(hull(X) + cone(V)) \cap \mathbb{Z}^n = \varnothing \iff (hull(X) + cone(V)) \cap \{-B, \ldots, B\}^n = \varnothing$$

Theorem

$B = (max(\{1\} \cup \{\|v\| \mid v \text{ is row vector of } A\}))^n$

Gomory Cuts

Definition (Cut)

given solution α to problem over $\mathbb{R}^n$, cut is inequality $a_1 x_1 + \cdots + a_n x_n \leqslant b$ which is not satisfied by α but by every $\mathbb{Z}^n$-solution

Gomory Cuts: Assumptions

- DPLL(T) Simplex returned solution α and final tableau A such that

$$A\vec{x}_N = \vec{x}_B \qquad\qquad -\infty \leqslant l_i \leqslant x_i \leqslant u_i \leqslant +\infty$$

- for some $i \in B$ have $\alpha(x_i) \notin \mathbb{Z}$ and for all $j \in N$ value $\alpha(x_j)$ is l_j or u_j

Notation

- write $c = \alpha(x_i) - \lfloor \alpha(x_i) \rfloor$
- $L^+ = \{\, j \in L \mid \alpha(x_j) = l_j \text{ and } A_{ij} \geqslant 0 \,\} \quad U^+ = \{\, j \in U \mid \alpha(x_j) = u_j \text{ and } A_{ij} \geqslant 0 \,\}$
  $L^- = \{\, j \in L \mid \alpha(x_j) = l_j \text{ and } A_{ij} < 0 \,\} \quad U^- = \{\, j \in U \mid \alpha(x_j) = u_j \text{ and } A_{ij} < 0 \,\}$

Lemma (Gomory Cut)

$$\sum_{j \in L^+} \frac{A_{ij}}{1-c}(x_j - l_j) - \sum_{j \in U^-} \frac{A_{ij}}{1-c}(u_j - x_j) - \sum_{j \in L^-} \frac{A_{ij}}{c}(x_j - l_j) + \sum_{j \in U^+} \frac{A_{ij}}{c}(u_j - x_j) \geqslant 1$$

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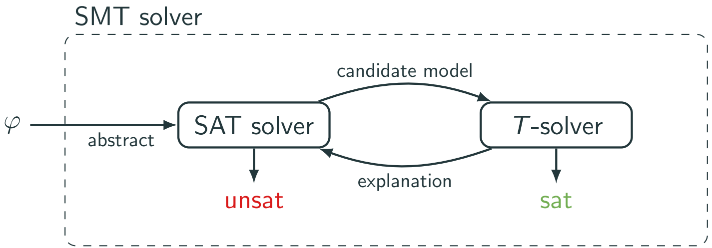
How to Be Lazy



Theory T	T-solving method	
▶ equality logic	equality graphs	✓
▶ equality + uninterpreted functions (EUF)	congruence closure	✓
▶ linear real arithmetic (LRA)	DPLL(T) Simplex	✓
▶ linear integer arithmetic (LIA)	DPLL(T) Simplex + cuts	✓
▶ bitvectors (BV)	bit-blasting	
▶ arrays (A)		

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\mathit{rel} &:= \ = \mid \neq \mid \geqslant_u \mid \geqslant_s \mid >_u \mid >_s \\
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- axioms are equality axioms plus rules for arithmetic/comparison/bitwise operations on bit vectors of length k
- solution assigns bit list of length k to variables $\mathbf{x}_k$

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  $\mathbf{x}[i{:}j]$ denotes $x_i \dots x_j$
  and :: is concatenation

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- $\mathbf{xn}_k$ is binary representation of hexadecimal n in k bits

Example

- $\mathbf{0}_1$, $\mathbf{3}_2$, $\mathbf{10}_4$, $\mathbf{1024}_{32}$,...
- $\mathbf{x0}_4$, $\mathbf{xa}_4$, $\mathbf{xb0}_8$, $\mathbf{x11cf}_{16}$,$\mathbf{xffffffff}_{32}$,...

More examples

negation uses two's complement

- $-\mathbf{a}_4 = \mathbf{a}_4$
  satisfiable: $v(\mathbf{a}_4) = -\mathbf{8}_4 = \mathbf{x8}_4$
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satisfied for powers of 2 (and 0)

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Definition (Bit Blasting: Formulas)

bit blasting transformation **B** transforms BV formula into propositional formula:

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$$\mathbf{B}(t_1 \ \mathit{rel} \ t_2) = \mathbf{B}_r(u_1 \ \mathit{rel} \ u_2) \wedge \varphi_1 \wedge \varphi_2 \quad \text{if } \mathbf{B}_t(t_1) = (u_1, \varphi_1) \text{ and } \mathbf{B}_t(t_2) = (u_2, \varphi_2)$$

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$\mathbf{B}_r$ transforms atom into propositional formula

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bit blasting $\mathbf{B}_t$ for term t returns (result u, side condition φ)

$$\mathbf{B}(t_1 \; rel \; t_2) = \mathbf{B}_r(u_1 \; rel \; u_2) \wedge \varphi_1 \wedge \varphi_2 \quad \text{if } \mathbf{B}_t(t_1) = (u_1, \varphi_1) \text{ and } \mathbf{B}_t(t_2) = (u_2, \varphi_2)$$

Definition (Bit Blasting: Atoms)
for bit vectors $\mathbf{x}_k$ and $\mathbf{y}_k$ set

- equality

$$\mathbf{B}_r(\mathbf{x}_{k+1} = \mathbf{y}_{k+1}) = (x_k \leftrightarrow y_k) \wedge \cdots \wedge (x_1 \leftrightarrow y_1) \wedge (x_0 \leftrightarrow y_0)$$

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$$\mathbf{B}_r(\mathbf{x}_{k+1} \neq \mathbf{y}_{k+1}) = (x_k \oplus y_k) \vee \cdots \vee (x_1 \oplus y_1) \vee (x_0 \oplus y_0)$$

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- unsigned greater-than

$$\mathbf{B}(\mathbf{x}_k >_u \mathbf{y}_k) = \mathbf{B}(\mathbf{x}_k \geqslant \mathbf{y}_k) \land \mathbf{B}(\mathbf{x}_k \neq \mathbf{y}_k)$$

Definition (Bit Blasting: Bitwise Operations)
for bit vectors $\mathbf{x}_k$ and $\mathbf{y}_k$ use fresh variable $\mathbf{z}_k$ and set

- bitwise and

$$\mathbf{B}_t(\mathbf{x}_k \mathbin{\&} \mathbf{y}_k) = (\mathbf{z}_k, \varphi) \qquad \varphi = \bigwedge_{i=0}^{k-1} z_i \leftrightarrow (x_i \wedge y_i)$$

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- bitwise negation

$$\mathbf{B}_t(-\mathbf{x}_k) = (\mathbf{z}_k, \varphi) \qquad \varphi = \bigwedge_{i=0}^{k-1} z_i \leftrightarrow \neg x_i$$

Definition (Bit Blasting: Concatenation, Extraction, If)

- concatenation

  $$\mathbf{B}_t(\mathbf{x}_k :: \mathbf{y}_m) = (\mathbf{x}_k\mathbf{y}_m, \top)$$

  for bit vectors $\mathbf{x}_k$ and $\mathbf{y}_m$

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- extraction

$$\mathbf{B}_t(\mathbf{x}[n{:}m]) = (\mathbf{z}_{n-m+1}, \varphi) \quad \varphi = \bigwedge_{i=0}^{n-m} z_i \leftrightarrow x_{i+m}$$

  for bit vector $\mathbf{x}_k$, $k > n \geqslant m \geqslant 0$ and fresh variable $\mathbf{z}_{n-m+1}$

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  for bit vector $\mathbf{x}_k$, $k > n \geqslant m \geqslant 0$ and fresh variable $\mathbf{z}_{n-m+1}$

- if-then-else

$$\mathbf{B}_t(p \mathbin{?} \mathbf{x}_k : \mathbf{y}_k) = (\mathbf{z}_k, \varphi) \qquad \varphi = \bigwedge_{i=0}^{k-1} (p \rightarrow (z_i \leftrightarrow x_i)) \wedge (\neg p \rightarrow (z_i \leftrightarrow y_i))$$

  for formula p and bit vectors $\mathbf{x}_k$ and $\mathbf{y}_k$

Definition (Bit Blasting: Addition and Subtraction)

- addition

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for fresh variables $\mathbf{s}_k$ and $\mathbf{c}_k$ and $\mathrm{min2}(a, b, d) = (a \wedge b) \vee (a \wedge d) \vee (b \wedge d)$

ripple-carry adder:
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for fresh variables $\mathbf{s}_k$ and $\mathbf{c}_k$ and $\mathrm{min2}(a, b, d) = (a \wedge b) \vee (a \wedge d) \vee (b \wedge d)$

- unary minus

$$\mathbf{B}_t(-\mathbf{x}_k) = \mathbf{B}_t(\sim \mathbf{x}_k + \mathbf{1}_k)$$

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ripple-carry adder: $\mathbf{c}_k$ are carry bits

where

$$\varphi = (c_0 \leftrightarrow x_0 \wedge y_0) \wedge (s_0 \leftrightarrow x_0 \oplus y_0) \wedge \bigwedge_{i=1}^{k-1} (c_i \leftrightarrow \mathrm{min2}(x_i, y_i, c_{i-1})) \wedge (s_i \leftrightarrow x_i \oplus y_i \oplus c_{i-1})$$

for fresh variables $\mathbf{s}_k$ and $\mathbf{c}_k$ and $\mathrm{min2}(a, b, d) = (a \wedge b) \vee (a \wedge d) \vee (b \wedge d)$

- unary minus

$$\mathbf{B}_t(-\mathbf{x}_k) = \mathbf{B}_t(\sim \mathbf{x}_k + \mathbf{1}_k)$$

- subtraction

$$\mathbf{B}_t(\mathbf{x}_k - \mathbf{y}_k) = \mathbf{B}_t(\mathbf{x}_k + (-\mathbf{y}_k))$$

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for bit vectors $\mathbf{x}_k$ and $\mathbf{y}_k$ set

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- unsigned division

$$\mathbf{B}_t(\mathbf{x}_k \div_u \mathbf{y}_k) = (\mathbf{q}_k, \varphi)$$

$$\varphi = \mathbf{B}(\mathbf{y}_k \neq \mathbf{0}_k \to (\mathbf{q}_k \times \mathbf{y}_k + \mathbf{r}_k = \mathbf{x}_k \land \mathbf{r}_k < \mathbf{y}_k \land \mathbf{q}_k < \mathbf{x}_k))$$

for fresh variables $\mathbf{q}_k$ and $\mathbf{r}_k$

Example (SMT-LIB 2 for BV)

$(\mathbf{a}_4 + \mathbf{b}_4 <_u \mathbf{b}_4) \wedge (\mathbf{a}_4 \neq \mathbf{10}_4) \wedge (\mathbf{a}_4 \,\&\, \mathbf{b}_4 = \mathbf{8}_4)$ is expressed as

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- `bvand`, `bvor` are bitwise logical operations

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LLVM

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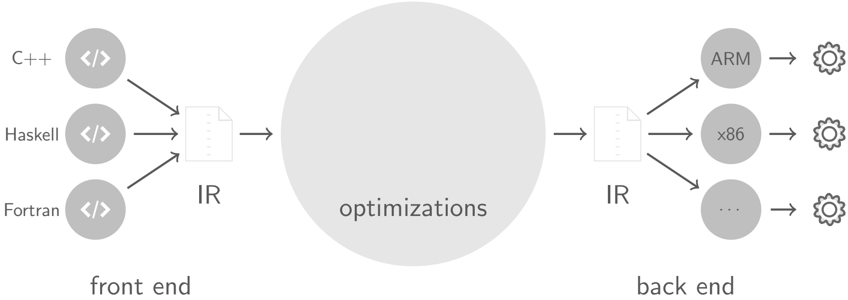

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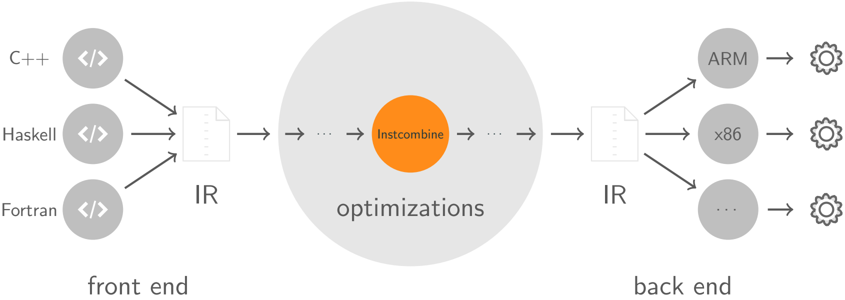

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Instcombine Pass

- over 1000 algebraic simplifications of expressions
  - transform multiplies with constant power-of-two argument into shifts
  - bitwise operators with constant operands are always grouped so that shifts are performed first, then ors, then ands, then xors
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  - ...

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Example

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int foo(int z) {
int x = 4 * (z | 1001);
return -256 & x;
}
```

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define i32 @foo(i32) #0 {
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  - ...
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- sometimes optimizations have errors—and compiler bugs are critical

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Alive Project

- represent Instcombine optimizations in domain-specific language, e.g.

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(assert (= before (bvsub #x00000000 (bvsdiv x c))))
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(assert (not (= before after)))
(assert (not (= c #x00000000)))
(check-sat)
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- wrong for `c = x = #x80000000`

Bit Vectors in python/z3

```
from z3 import *
x = BitVec('x', 32) # create variable named x with 32 bits
c = BitVec('c', 32)

before = BitVecVal(0, 32) - (x / c)
after = x / - c

solver = Solver()
solver.add(c != BitVecVal(0, 32)) # exclude case where c=0
solver.add(after != before)

result = solver.check()
if result == z3.sat:
  m = solver.model()
  print m[x], m[c] # 2147483648 2147483648
  print m.eval(before), m.eval(after) # 4294967295 1
```

Application: Detecting Nontermination

```
int bsearch(int a[], int k, unsigned int lo, unsigned int hi) {
  unsigned int mid;
  while (lo < hi) {
    mid = (lo + hi)/2;
    if (a[mid] < k)
      lo = mid + 1;
    else if (a[mid] > k)
      hi = mid - 1;
    else
      return mid;
  }
  return -1;
}
```

- (former) implementation of binary search in Java library
- loops for inputs `lo=1` and `hi=UINT_MAX` if `a[0] < k`.
- SMT encoding can find values such that parameters stay the same in recursive call

Bibliography

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