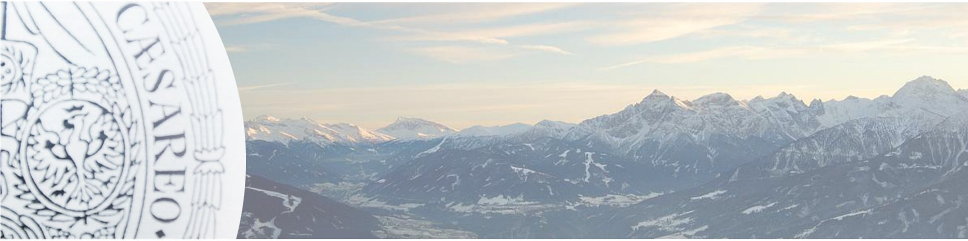

SAT and SMT Solving

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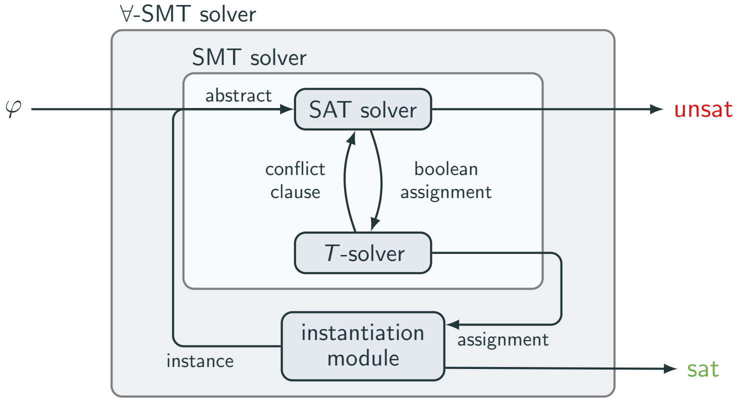
lecture 13
SS 2019

Outline

- Summary of Last Week
- Bounded Model Checking for Verification
- Test
- Evaluation
- More on SAT and SMT

Instantiation Framework



- split φ into
  - literals φ_Q with quantifiers
  - literals φ_E without quantifiers
- instantiation module generates instances of φ_Q to extend φ_E

SMT solver is in general no decision procedure in presence of ∀ quantifiers

Skolemization

1. bring formula into prenex form
2. replace $\forall x_1, \dots, x_k \exists y\ \psi[y]$ by $\forall x_1, \dots, x_k\ \psi[\mathsf{f}(x_1, \dots, x_k)]$ for fresh f
   until no existential quantifiers left

Theorem

can consider formulas of shape $\forall x_1, \dots, x_n\ \varphi[x_1, \dots, x_n]$

if φ' is skolemization of φ then φ and φ' are equisatisfiable

Instantiation via E-matching

for each $\forall \overline{x}.\psi$

- select set of instantiation patterns $\{t_1, \dots, t_n\}$
- for each t_i let S_i be set of substitutions σ such that $t_i\sigma$ occurs in φ_E
- add $\{\psi\sigma \mid \sigma \in S_i\}$ to φ_E

Theorem (Stronger Herbrand)

$\varphi_E \wedge \varphi_Q$ is unsatisfiable if and only if there exist infinite series

- $\mathbf{E}_i$ *of finite literals sets*
- $\mathbf{Q}_i$ *of finite sets of φ_Q instances*

such that

- $\mathbf{Q}_i \subseteq \{\psi\sigma \mid \forall \overline{x}.\, \psi$ *occurs in* φ_Q *and* $\mathrm{dom}(\sigma) = \overline{x}$ *and* $\mathrm{ran}(\sigma) \subseteq \mathcal{T}(\mathbf{E}_i)\}$
- $\mathbf{E}_0 = \varphi_E$ *and* $\mathbf{E}_{i+1} = \mathbf{E}_i \cup \mathbf{Q}_i$
- *some* $\mathbf{E}_n$ *is unsatisfiable*

Instantiation via enumeration

Fix ordering $>$ on tuples of terms without quantified variables.

Given assignment $\mathbf{E}_i$ from T-solver

- for each $\forall \overline{x}.\psi$ in φ_Q
  - search minimal $\overline{x}\sigma$ with respect to $\succeq$ such that $\overline{x}\sigma \in \mathcal{T}(\mathbf{E}_i)$ and $\mathbf{E}_i \not\models \psi\sigma$
  - if exists, add $\{\psi\sigma\}$ to $\mathbf{Q}_i$

If $\mathbf{Q}_i = \varnothing$ then sat, otherwise return $\mathbf{Q}_i$

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Disastrous Software Bugs

Ariane 5 Flight 501 (1996)

- destroyed 37 seconds after launch
- software for Ariane 4 for was reused
- software error: data conversion from 64-bit floating point to 16-bit integer caused arithmetic overflow
- cost: 370 million $

http://en.wikipedia.org/wiki/Ariane_5_Flight_501

Mars Exploration Rover “Spirit” (2004)

- landed on January 4
- stopped communicating on January 21
- software error: stuck in reboot loop
- reboot failed because of flash memory failure, ultimate problem: too many files

http://en.wikipedia.org/wiki/Spirit_(rover)

Heathrow Terminal 5 Opening (2008)

- baggage system collapsed on opening day
- 42,000 bags not shipped with their owners, 500 flights cancelled
- software was tested but did not work properly with real-world load
- cost 50 million £

http://www.zdnet.com/article/it-failure-at-heathrow-t5-what-really-happened

Trading Glitch at Knight Capital (2012)

- bug in trading software resulted in 45 minutes of uncontrolled buys
- company did 11% of US trading that year
- software was run in invalid configuration
- 440 million $ lost

http://en.wikipedia.org/wiki/Knight_Capital_Group

Death in Self-Driving Car Crash (2018)

- person died in accident with Uber's self-driving car
- victim was wrongly classified by software as non-obstacle

http://www.siliconrepublic.com/companies/uber-bug-crash

Software is Ubiquitous in Critical Systems

transport, energy, medicine, communication, finance, embedded systems, ...

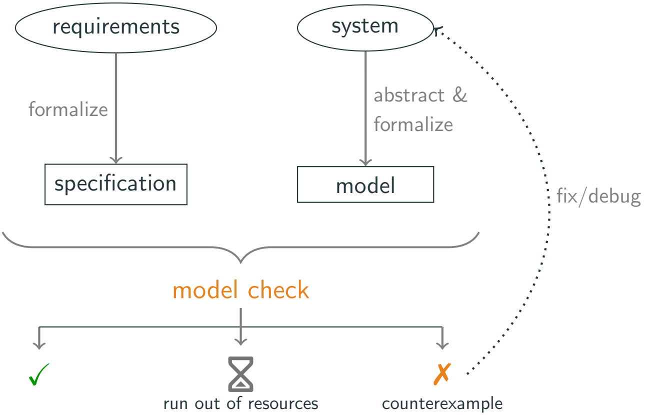
How to Ensure Correctness of Software?

- testing
  - \+ cheap, simple
  - – checks desired result only for given set of testcases
- verification
  - \+ can prove automatically that system meets specification, i.e., desired output is delivered for all inputs
  - – more costly

Model Checking

- widely used verification approach to
  - find bugs in software and hardware
  - prove correctness of models
- Turing Award 2007 for Clarke, Emerson, and Sifakis
- bounded model checking can be reduced to SAT/SMT

Model Checking: Workflow

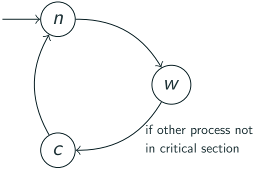


Model Checking Example: Mutex (1)

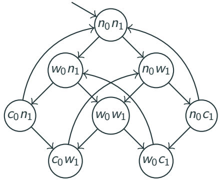
- concurrent processes P_0, P_1 share some resource, access controlled by mutex
- program run by P_0, P_1 matches pattern

```
# non-critical section
while (other process critical) :
  wait ()
# critical section
# non-critical section
```

- process can be abstracted to model $\mathcal{M} = \langle S, R \rangle$
with states $S = \{n, w, c\}$ and transitions R:



Model Checking Example: Mutex (2)



- obtain model for 2 processes by product construction: write $s_0 s_1$ for P_0 being in state s_0 and P_1 in state s_1
- desired properties:

safe: only one process is in its critical section at any time

live: whenever any process wants to enter its critical section, it will eventually be permitted to do so

non-blocking: a process can always request to enter its critical section

- how to formalize desired properties? temporal logic, e.g. LTL or CTL

safe: $\mathsf{G}\ \neg(c_0 \wedge c_1)$ ✓ as $c_0 c_1$ unreachable

live: $\mathsf{G}\ (w_0 \to \mathsf{F}\ c_0)$ ✗ e.g. 

non-blocking: $\mathsf{AG}\ (n_0 \to \mathsf{EX}\ w_0)$ ✓

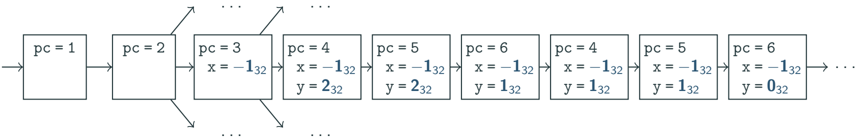
Common Kinds of Properties

- $\mathsf{G}\ \psi$ for propositional formula ψ is safety property
- $\mathsf{G}\ (\psi \to \mathsf{F}\chi)$ for propositional formulas ψ,χ is liveness property

Example: Can This Program Cause An Overflow? (1)

```
1 void main() {
2   int x = -1;
3   int y = input();
4   while (y<100) {
5     y = y+x;
6   }
7 }
```

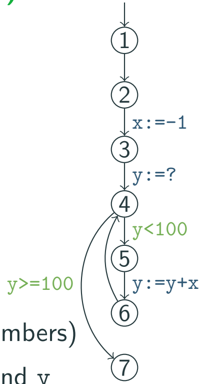
- model checking problem: addition $x + y$ in line 5 does not over/underflow
  - state is assignment of x, y + value of program counter pc
  - property $\mathsf{G}\ (\mathsf{pc} = 5 \rightarrow ((x > \mathbf{0}_{32} \wedge x + y > y) \vee (x \leqslant \mathbf{0}_{32} \wedge x + y \leqslant y)))$
- (part of) model:



- but state space is very large: $(2^{32})^2 \cdot 7$ for bit width 32
- cannot check all possible values

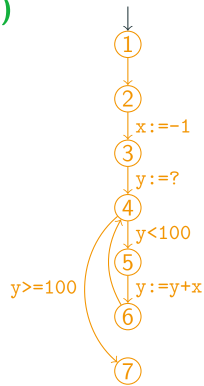
Example: Can This Program Cause An Overflow? (2)

```
1 void main() {
2   int x = -1;
3   int y = input();
4   while (y<100) {
5     y = y+x;
6   }
7 }
```



- construct program graph G
- $\{1, \ldots, 7\}$ are possible values of program counter (line numbers)
- state is tuple $\langle \mathrm{pc}, x, y \rangle$ of values of program counter, x, and y
- state of form $\langle 1, \ldots, \ldots \rangle$ is initial state
- examples of state transitions according to G:
  - $\langle 4, -\mathbf{1}_{32}, \mathbf{10}_{32} \rangle \to \langle 5, -\mathbf{1}_{32}, \mathbf{10}_{32} \rangle$ is possible
  - $\langle 4, -\mathbf{1}_{32}, \mathbf{101}_{32} \rangle \to \langle 7, -\mathbf{1}_{32}, \mathbf{101}_{32} \rangle$ is possible
  - $\langle 4, \mathbf{10}_{32}, \mathbf{101}_{32} \rangle \to \langle 5, \mathbf{10}_{32}, \mathbf{101}_{32} \rangle$ is not possible
  - $\langle 4, -\mathbf{1}_{32}, \mathbf{1}_{32} \rangle \to \langle 5, -\mathbf{1}_{32}, \mathbf{2}_{32} \rangle$ is not possible

Example: Can This Program Cause An Overflow? (3)



1. define predicates
   - $I(\langle pc, x, y\rangle) = (pc = 1)$ to characterize initial state
   - to characterize possible state transitions:

$$
\begin{aligned}
&T(\langle pc, x, y\rangle, \langle pc', x', y'\rangle) = \\
&\quad (pc = 1 \wedge pc' = 2) \vee (pc = 2 \wedge pc' = 3 \wedge x' = -1) \vee \\
&\quad (pc = 3 \wedge pc' = 4 \wedge x = x') \vee \\
&\quad (pc = 4 \wedge pc' = 5 \wedge y < 100 \wedge x = x' \wedge y = y') \vee \\
&\quad (pc = 5 \wedge pc' = 6 \wedge y' = y + x \wedge x = x') \vee \\
&\quad (pc = 4 \wedge pc' = 7 \wedge y \geqslant 100 \wedge x = x' \wedge y = y') \vee \\
&\quad (pc = 6 \wedge pc' = 4 \wedge x = x' \wedge y = y')
\end{aligned}
$$

   - $P(\langle pc, x, y\rangle) = (pc = 5) \wedge ((x > \mathbf{0}_{32} \wedge x + y \leqslant y) \vee (x \leqslant \mathbf{0}_{32} \wedge (y + x > y)))$

2. for states $s_0, \ldots, s_k$ formula φ_k expresses overflow occurring within k steps:

$$\varphi_k = I(s_0) \wedge \bigwedge_{i=0}^{k-1} T(s_i, s_{i+1}) \wedge \bigvee_{i=0}^{k} P(s_i)$$

3. if φ_k satisfiable then overflow can occur within k steps, e.g. for $k = 5$

Bounded Model Checking

- find counterexamples to desired property of transition system (bugs)
- counterexamples are bounded in size

Definition (Transition System)

transition system $\mathcal{T} = (S, \rightarrow, S_0, L)$ where

- S is set of states
- $\rightarrow \subseteq S \times S$ is transition relation
- $S_0 \subseteq S$ is set of initial states
- A is a set of propositional atoms
- $L : S \rightarrow 2^A$ is labeling function associating state with subset of A

Remark

S and A may be (countably) infinite

Bounded Model Checking: Safety Properties

Idea

given transition system and property $\mathsf{G}\ \psi$, look for counterexamples in $\leqslant k$ steps

$$\begin{array}{ccccccccc} \to s_0 & \to & s_1 & \to & s_2 & \to & \cdots & \to & s_k \\ \neg\psi & \vee & \neg\psi & \vee & \neg\psi & \vee & \cdots & \vee & \neg\psi \end{array}$$

SAT/SMT Encoding

given transition system $\mathcal{T}$ and safety property $\mathsf{G}\ \psi$

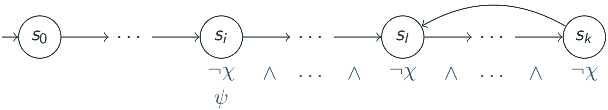
- use encoding $\langle s \rangle$ of state $s \in S$ by set of SAT/SMT variables
- use predicates
  - I for initial states such that use $I(\langle s \rangle)$ is true iff $s \in S_0$
  - T for transitions such that $T(\langle s \rangle, \langle s' \rangle)$ is true iff $s \to s'$ in $\mathcal{T}$
  - P such that $P(\langle s \rangle)$ is true iff ψ holds in s
- use different fresh variables for $k+1$ states $\langle s_0 \rangle, \ldots, \langle s_k \rangle$
- check satisfiability of

$$I(\langle s_0 \rangle) \wedge \bigwedge_{i=0}^{k-1} T(\langle s_i \rangle, \langle s_{i+1} \rangle) \wedge \bigvee_{i=0}^{k} \neg P(\langle s_i \rangle)$$

Bounded Model Checking: Liveness Properties

Idea

- counterexample to liveness property $\mathsf{G}\,(\psi \to \mathsf{F}\chi)$ requires infinite path
- look for counterexamples in $\leqslant k$ steps of lasso shape:



SAT/SMT Encoding

given transition system $\mathcal{T}$ and liveness property $\mathsf{G}\,(\psi \to \mathsf{F}\chi)$

- use encoding of states, predicates I and T as for safety properties
- predicate P such that $P(\langle s \rangle)$ is true iff ψ holds in s
- predicate C such that $C(\langle s \rangle)$ is true iff χ holds in s
- check satisfiability of

$$I(\langle s_0 \rangle) \wedge \bigwedge_{i=0}^{k-1} T(\langle s_i \rangle, \langle s_{i+1} \rangle) \wedge \bigvee_{i=0}^{k} \left(P(\langle s_i \rangle) \wedge \bigwedge_{j=i}^{k} \neg C(\langle s_i \rangle) \wedge \bigvee_{l=i}^{k} T(\langle s_k \rangle, \langle s_l \rangle) \right)$$

Transition System $\mathcal{T}(P)$ from Program P

- state $\langle pc, v_0, \ldots, v_n \rangle$ consists of
  - value for program counter pc, i.e. line number in P
  - assignment for variables in scope $\mathrm{v}_0, \ldots, \mathrm{v}_n$
- there is step $s \to s'$ for $s = \langle pc, v_0, \ldots, v_n \rangle$ and $s' = \langle pc', v_0', \ldots, v_n' \rangle$ iff P admits step from s to s'
- S_0 consists of initial program states
- atom set A consists of all propositional formulas over $pc, v_0, \ldots, v_n$
- labeling $L(s)$ is set of all atoms A which hold in $s = \langle pc, v_0, \ldots, v_n \rangle$

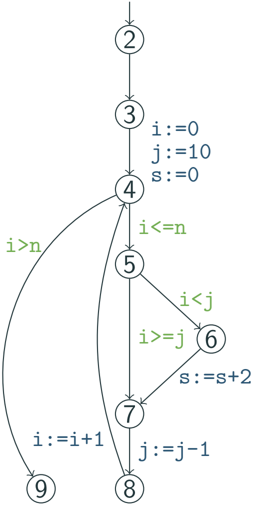
Program Graph

- nodes are line numbers
- edge from line l to line l' if program counter can go from line l to l'
- two kinds of edge labels:
  - conditions for program counter to take this path
  - assignments of updated variables
- program graph is useful to derive encoding of $\mathcal{T}(P)$

Checking an Explicit Assertion

```
1  int n;
2  int main() {
3    int i=0, j=10, s=0;
4    for(i=0; i<=n; i++) {
5      if (i<j)
6        s = s + 2;
7      j--;
8    }
9    assert(s==n*2 || s == 0);
10 }
```

- construct program graph
- states are of form $\langle \mathrm{pc}, i, j, n, s \rangle$
- safety property to check is $\mathrm{G}\ (\mathrm{pc} = 9 \rightarrow (s = 2n \vee s = 0))$
- see `verification.py`



Software Verification Competition (SV-COMP)

- annual competition
  `https://sv-comp.sosy-lab.org/2018/`
- industrial (and crafted) benchmarks
  `https://github.com/sosy-lab/sv-benchmarks`
- many tools use SMT solvers

Common Safety Properties

- no overflow in addition: $(\mathtt{x} > 0 \wedge \mathtt{x} + \mathtt{y} \geqslant \mathtt{y}) \vee (\mathtt{x} \leqslant 0 \wedge \mathtt{x} + \mathtt{y} \leqslant \mathtt{y})$
- array accesses in bounds: $0 \leqslant \mathtt{i} < \mathit{size}(\mathtt{a})$ for all accesses `a[i]`
- memory safety: set predicate $\mathit{ok}(\mathit{addr})$ when memory allocated, check $\mathit{ok}(\mathtt{p})$ for every dereference $*\mathtt{p}$
- explicit assertions

Bibliography

Armin Biere, Alessandro Cimatti, Edmund M. Clarke, Ofer Strichman, and Yunshan Zhu.
Bounded Model Checking
Advances in Computers 58, pp 117–148, 2003.

Armin Biere.
Bounded Model Checking.
Chapter 14 in: Handbook of Satisfiability, IOS Press, pp. 457–481, 2009.

Test

- July 1, 14:00, HSB 9 (with René Thiemann)
- material includes everything up to week 12
- 60 minutes
- see test of last year

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Evaluation

- **LV-Code: 703048**
- **additional questions**
  (a) There was too much theory and too little about applications in the course content.
  (b) In my opinion it makes sense that computer science students learn about using SAT/SMT to solve constraint problems.
  (c) I think I might use a SAT/SMT solver in the future.
  (d) I would prefer a different programming language than Python.

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Upcoming Courses

Computational Logic

- exciting course about theorem proving
- in summer term 2020 taught by Cezary Kaliszyk and Vincent van Oostrom

Advanced Topics in Term Rewriting

- exciting course about special topics in term rewriting
- with some applications of SAT/SMT therein
- in winter term 2019 taught by Aart Middeldorp

Available Bachelor Projects

Fast Multiset Comparisons

- multisets are data structures appearing in many applications
- comparing two multisets is NP-complete problem
- aim is to implement fast algorithm based on verified SAT solver

Maximal Interpretations

- investigate applications of maxSAT/maxSMT in termination methods for term rewrite systems
- implement in Tyrolean Complexity Tool

`http://cl-informatik.uibk.ac.at/teaching/smb/available.php?q=%23Logic`


CAT