ATP - SS2020

May 26, 2020

Solutions - week 1

Stating the Robbins conjecture requires a binary operation that is commutative and associative and then saying that the Huntington equation is equivalent to the Robbins equation. Namely.

```
fof(commutative, axiom, ![X,Y]: plus(X,Y) = plus(Y,X) ).
fof(associative, axiom, ![X,Y,Z]: plus(plus(X,Y),Z) = plus(X,plus(Y,Z)) ).
fof(hunt_rob, conjecture, ! [X,Y] : ((plus(n(plus(n(X),Y)),n(plus(n(X),n(Y)))) = X)
     <=> (n(plus(n(plus(X,Y)),n(plus(X,n(Y)))))=X))).
```

Note that this is quite a hard problem and by itself is too complicated to prove in one go. But some of you have tried the implication only (Robbins implies Huntington):

```
fof(commutative, axiom, ![X,Y]: plus(X,Y) = plus(Y,X) ).
fof(associative, axiom, ![X,Y,Z]: plus(plus(X,Y),Z) = plus(X,plus(Y,Z)) ).
fof(robbins, axiom, ! [X,Y] : (plus(n(plus(n(X),Y)),n(plus(n(X),n(Y)))) = X)).
fof(hunt, conjecture, ! [X,Y]: (n(plus(n(plus(X,Y)),n(plus(X,n(Y)))))=X)).
```

and that works.

```
eprover -s -p --tstp-format --auto rr.p
```

Gives:

```
fof(c_0_0, axiom, (![X1]:![X2]:![X3]:plus(plus(X1,X2),X3)=plus(X1,plus(X2,X3))), file('rr.p', ass
fof(c_0_1, axiom, (![X1]:![X2]:plus(n(plus(n(X1),X2)),n(plus(n(X1),n(X2))))=X1), file('rr.p', rob
fof(c_0_2, axiom, (![X1]:![X2]:plus(X1,X2)=plus(X2,X1)), file('rr.p', commutative)).
fof(c_0_3, conjecture, (![X1]:![X2]:n(plus(n(plus(X1,X2)),n(plus(X1,n(X2)))))=X1), file('rr.p', h
fof(c_0_4, plain, (![X4]:![X5]:![X6]:plus(plus(X4,X5),X6)=plus(X4,plus(X5,X6))), inference(variab
fof(c_0_5, plain, (![X3]:![X4]:plus(n(plus(n(X3),X4)),n(plus(n(X3),n(X4))))=X3), inference(variab
fof(c_0_6, plain, (![X3]:![X4]:plus(X3,X4)=plus(X4,X3)), inference(variable_rename,[status(thm)],
cnf(c_0_7, plain, (plus(plus(X1,X2),X3)=plus(X1,plus(X2,X3))), inference(split_conjunct,[status(t
cnf(c_0_8, plain, (plus(n(plus(n(X1),X2)),n(plus(n(X1),n(X2))))=X1), inference(split_conjunct,[st
cnf(c_0_9, plain, (plus(X1,X2)=plus(X2,X1)), inference(split_conjunct,[status(thm)],[c_0_6])).
cnf(c_0_10, plain, (plus(n(plus(n(X1),X2)),plus(n(plus(n(X1),n(X2))),X3))=plus(X1,X3)), inference
cnf(c_0_11, plain, (plus(X1,plus(X2,X3))=plus(X2,plus(X1,X3))), inference(rw,[status(thm)],[infer
cnf(c_0_12, plain, (plus(X1,n(plus(n(X1),n(n(X2)))))=plus(X1,n(plus(n(X1),X2)))), inference(rw,[s
cnf(c_0_13, plain, (plus(X1,plus(X2,n(plus(n(X1),n(n(X3))))))=plus(X2,plus(X1,n(plus(n(X1),X3))))
cnf(c_0_14, plain, (plus(n(plus(n(X1),X2)),n(plus(n(X2),n(X1))))=X1), inference(spm,[status(thm)]
cnf(c_0_15, plain, (plus(X1,plus(X2,X3))=plus(X3,plus(X1,X2))), inference(spm,[status(thm)],[c_0_
cnf(c_0_16, plain, (plus(X1,plus(n(plus(n(X1),X2)),n(plus(n(n(X2)),X1))))=plus(X1,n(X2))), infere
cnf(c_0_17, plain, (plus(n(plus(X1,n(X2))),n(plus(n(X1),n(X2))))=X2), inference(spm,[status(thm)]
cnf(c_0_18, plain, (plus(n(X1),n(n(X1)))=plus(X1,n(X1))), inference(rw,[status(thm)],[inference(s
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fof(c_0_19, negated_conjecture, (~(![X1]:![X2]:n(plus(n(plus(X1,X2)),n(plus(X1,n(X2)))))=X1)), in
cnf(c_0_20, plain, (plus(n(plus(X1,n(X1))),n(plus(X1,n(n(X1)))))=n(X1)), inference(rw,[status(thm
fof(c_0_21, negated_conjecture, (n(plus(n(plus(esk1_0,esk2_0)),n(plus(esk1_0,n(esk2_0)))))!=esk1_
cnf(c_0_22, plain, (n(n(X1))=X1), inference(spm,[status(thm)],[c_0_8, c_0_20])).
cnf(c_0_23, negated_conjecture, (n(plus(n(plus(esk1_0,esk2_0)),n(plus(esk1_0,n(esk2_0)))))!=esk1_
cnf(c_0_24, plain, (plus(n(plus(X1,X2)),n(plus(X1,n(X2))))=n(X1)), inference(spm,[status(thm)],[c
cnf(c_0_25, negated_conjecture, ($false), inference(cn,[status(thm)],[inference(rw,[status(thm)],
# SZS output end CNFRefutation
# Training examples: 0 positive, 0 negative
```