Computational Logic	WS 2015/2016	LVA 703607
EXAM 1		March 4, 2016
name:		immatriculation number:

This exam consists of six exercises. The available points for each item are written in the margin. You need at least 50 points to pass.

15 **1** Complete the following table:

formula	$\alpha/\beta/\gamma/\delta$	existential	**AE**	valid
$A \supset B$		✓		×
$(\forall x)[(\exists y)R(x,y) \supset \neg(\exists z)R(z,f(y))]$				
$\neg(A \supset \neg(B \vee A))$		✓		
$(\forall x)P(x) \supset (\neg(\exists y)Q(y) \supset (\forall x)P(x))$	β			
$(\exists x)(\forall y)R(x,y) \supset (\forall z)(\exists w)R(w,z)$			✓	

2 Peirce's law states that the propositional formula $(((P \supset Q) \supset P) \supset P)$ is valid.

5 (a) Give a tableau proof of Peirce's law.

10 (b) Give a proof of Peirce's law in the Hilbert system with the axioms

1	$X \supset (Y \supset X)$	2	$(X \supset (Y \supset Z)) \supset ((X \supset Y) \supset (X \supset Z))$
3	$\bot \supset X$	4	$X \supset \top$
5	$\neg\neg X \supset X$	6	$X \supset (\neg X \supset Y)$
7	$\alpha \supset \alpha_1$	8	$\alpha \supset \alpha_2$
9	$(\beta_1 \supset X) \supset ((\beta_2 \supset X) \supset (\beta \supset X))$		

and Modus Ponens as only rule of inference.

10 (c) Give a tableau proof of the sentence

$$(\forall x)[P(x) \supset (\exists y)Q(y)] \supset (\forall x)(\exists y)[P(x) \supset Q(y)]$$

9 **3** Answer **three** of the following five questions.

- What is a *propositional consistency property*?
- State the *deduction theorem* for Hilbert systems.
- What is a *first-order Hintikka set*?
- State the *compactness theorem* for first-order logic.
- What is an *explicit definition* of an n-place relation symbol R with respect to a set S of sentences?

4 This exercise is about Craig's interpolation theorem.

10 (a) Consider the following tableau proof of $\Phi = [A \wedge (B \vee C)] \supset \neg[(A \vee D) \supset \neg(\neg B \supset C)]$:

$$\neg\Phi$$

$$A \wedge (B \vee C)$$

$$\neg\neg[(A \vee D) \supset \neg(\neg B \supset C)]$$

$$(A \vee D) \supset \neg(\neg B \supset C)$$

$$A$$

$$B \vee C$$

$$\begin{array}{cc} \neg(A \vee D) & \neg(\neg B \supset C) \\ \neg A & \neg B \\ \neg D & \neg C \\ & \begin{array}{cc} B & C \end{array} \end{array}$$

Turn the tableau into a closed biased tableau and use the calculation rules for interpolants to compute an interpolant of Φ.

6 (b) There are four interpolation calculation rules for γ-formulas. Two of them are stated below:

$$\frac{S \cup \{L(\gamma(c))\} \xrightarrow{\text{int}} A}{S \cup \{L(\gamma)\} \xrightarrow{\text{int}} A} \quad \text{if constant } c \text{ does occur in } \{X_1, \ldots, X_n\}$$

$$\frac{S \cup \{L(\gamma(c))\} \xrightarrow{\text{int}} A}{S \cup \{L(\gamma)\} \xrightarrow{\text{int}} (\forall x)A\{c/x\}} \quad \text{if constant } c \text{ does not occur in } \{X_1, \ldots, X_n\}$$

Here $S = \{L(X_1), \ldots, L(X_n), R(Y_1), \ldots, R(Y_k)\}$. Give the other two.

5 This exercise is about Herbrand's theorem.

5 (a) Give the definition of validity functional form.

10 (b) Compute the Herbrand expansion of the sentence $(\exists x)(\forall y)[R(x,y) \supset \neg R(c, f(y))]$ over the Herbrand domain $D = \{c, f(c)\}$.

10 (c) Compute a tautologous Herbrand expansion for the sentence $(\exists x)[P(x) \supset (\forall y)P(y)]$.

10 **6** Determine whether the following statements are true or false. Every correct answer is worth 2 points. For every wrong answer 1 point is subtracted, provided the total number of points is non-negative.

true	false	statement
☐	☐	The propositional connective $\wedge$ is the dual of $\vee$.
☐	☐	In an **AE** tableau no δ-rule is applied after a γ-rule application.
☐	☐	The rule $\dfrac{\Gamma \to \Delta, X, Y}{\Gamma \to \Delta, X \vee Y}$ is an inference rule of sequent calculus.
☐	☐	A finite set S of formulas is Craig consistent if $\langle S_1 \rangle \supset \neg\langle S_2 \rangle$ has an interpolant for some partition $S_1 \uplus S_2$ of S.
☐	☐	A propositional consistency property $\mathcal{C}$ is of finite character provided $S \in \mathcal{C}$ if and only if every finite subset of S belongs to $\mathcal{C}$.