



name:

immatriculation number:

This exam consists of six exercises. The available points for each item are written in the margin. You need at least 50 points to pass.

- 15 1 Complete the following table:

formula	$\alpha/\beta/\gamma/\delta$	existential	<b>AE</b>	valid
$A \supset B$		✓		×
$(\forall x)[(\exists y)R(x, y) \supset \neg(\exists z)R(z, f(y))]$				
$\neg(A \supset \neg(B \vee A))$		✓		
$(\forall x)P(x) \supset (\neg(\exists y)Q(y) \supset (\forall x)P(x))$	$\beta$			
$(\exists x)(\forall y)R(x, y) \supset (\forall z)(\exists w)R(w, z)$			✓	

- 2 Peirce's law states that the propositional formula  $((P \supset Q) \supset P) \supset P$  is valid.

- 5 (a) Give a tableau proof of Peirce's law.  
 10 (b) Give a proof of Peirce's law in the Hilbert system with the axioms

- |   |   |
|---|---|
| 1 $X \supset (Y \supset X)$   | 2 $(X \supset (Y \supset Z)) \supset ((X \supset Y) \supset (X \supset Z))$ |
| 3 $\perp \supset X$   | 4 $X \supset \top$  |
| 5 $\neg\neg X \supset X$  | 6 $X \supset (\neg X \supset Y)$  |
| 7 $\alpha \supset \alpha_1$   | 8 $\alpha \supset \alpha_2$   |
| 9 $(\beta_1 \supset X) \supset ((\beta_2 \supset X) \supset (\beta \supset X))$ |   |

and Modus Ponens as only rule of inference.

- 10 (c) Give a tableau proof of the sentence

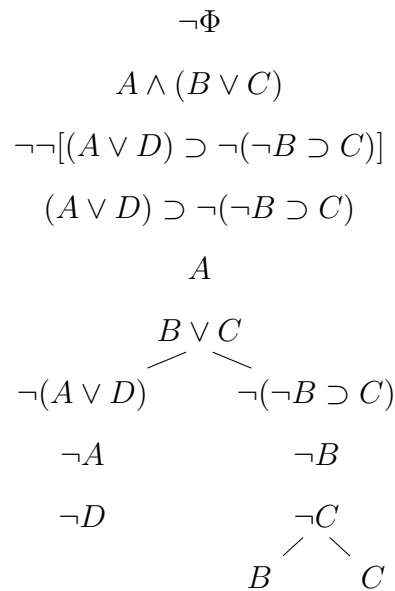
$$(\forall x)[P(x) \supset (\exists y)Q(y)] \supset (\forall x)(\exists y)[P(x) \supset Q(y)]$$

9 3 Answer three of the following five questions.

- What is a *propositional consistency property*?
- State the *deduction theorem* for Hilbert systems.
- What is a *first-order Hintikka set*?
- State the *compactness theorem* for first-order logic.
- What is an *explicit definition* of an  $n$ -place relation symbol  $R$  with respect to a set  $S$  of sentences?

4 This exercise is about Craig's interpolation theorem.

10 (a) Consider the following tableau proof of  $\Phi = [A \wedge (B \vee C)] \supset \neg[(A \vee D) \supset \neg(\neg B \supset C)]$ :



Turn the tableau into a closed biased tableau and use the calculation rules for interpolants to compute an interpolant of  $\Phi$ .

6 (b) There are four interpolation calculation rules for  $\gamma$ -formulas. Two of them are stated below:

$$\frac{S \cup \{L(\gamma(c))\} \xrightarrow{\text{int}} A}{S \cup \{L(\gamma)\} \xrightarrow{\text{int}} A} \quad \text{if constant } c \text{ does occur in } \{X_1, \dots, X_n\}$$

$$\frac{S \cup \{L(\gamma(c))\} \xrightarrow{\text{int}} A}{S \cup \{L(\gamma)\} \xrightarrow{\text{int}} (\forall x)A\{c/x\}} \quad \text{if constant } c \text{ does not occur in } \{X_1, \dots, X_n\}$$

Here  $S = \{L(X_1), \dots, L(X_n), R(Y_1), \dots, R(Y_k)\}$ . Give the other two.

- 5 5 This exercise is about Herbrand's theorem.
- 5 (a) Give the definition of validity functional form.
- 10 (b) Compute the Herbrand expansion of the sentence  $(\exists x)(\forall y)[R(x, y) \supset \neg R(c, f(y))]$  over the Herbrand domain  $D = \{c, f(c)\}$ .
- 10 (c) Compute a tautologous Herbrand expansion for the sentence  $(\exists x)[P(x) \supset (\forall y)P(y)]$ .
- 10 6 Determine whether the following statements are true or false. Every correct answer is worth 2 points. For every wrong answer 1 point is subtracted, provided the total number of points is non-negative.

true	false	statement
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- |                          |                          |   |
|--------------------------|--------------------------|---|
| <input type="checkbox"/> | <input type="checkbox"/> | The propositional connective $\wedge$ is the dual of $\vee$ .   |
| <input type="checkbox"/> | <input type="checkbox"/> | In an <b>AE</b> tableau no $\delta$ -rule is applied after a $\gamma$ -rule application.  |
| <input type="checkbox"/> | <input type="checkbox"/> | The rule $\frac{\Gamma \rightarrow \Delta, X, Y}{\Gamma \rightarrow \Delta, X \vee Y}$ is an inference rule of sequent calculus.  |
| <input type="checkbox"/> | <input type="checkbox"/> | A finite set $S$ of formulas is Craig consistent if $\langle S_1 \rangle \supset \neg \langle S_2 \rangle$ has an interpolant for some partition $S_1 \uplus S_2$ of $S$ .  |
| <input type="checkbox"/> | <input type="checkbox"/> | A propositional consistency property $\mathcal{C}$ is of finite character provided $S \in \mathcal{C}$ if and only if every finite subset of $S$ belongs to $\mathcal{C}$ . |