universität innsbruck
(Second





Computational Logic

WS 2015/2016

LVA 703607

EXAM 2

September 30, 2016

name:

immatriculation number:

This exam consists of six exercises. The available points for each item are written in the margin. You need at least 50 points to pass.

12 1 Complete the following table:

formula	$lpha / eta / \gamma / \delta$	universal	satisfiable
$\bot \supset (\forall x) P(x)$			\checkmark
$\neg(((A \supset B) \supset A) \supset A)$		\checkmark	
$(\forall x)P(x) \supset (\neg(\exists y)Q(y) \supset (\forall x)P(x))$			
$(\forall x)[(P(x) \supset Q(x)) \uparrow (\neg Q(x) \supset P(x))]$	γ		

2 Give tableau proofs of the following sentences.

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(a) $(\neg P \supset Q) \supset ((P \supset Q) \supset Q)$

10 (b) $(\forall x)[P(x) \lor Q(x)] \supset [(\forall x)P(x) \lor (\exists x)Q(x)]$

10 (c) $(\forall x)(\exists y)[P(x) \supset Q(y)] \supset (\forall x)[P(x) \supset (\exists y)Q(y)]$

9 3 Answer **three** of the following five questions.

- State and prove *Hintikka's lemma* for propositional logic.
- State Lyndon's homomorphism theorem.
- State at least seven axiom schemes of Hilbert systems.
- What is an *interpolant* for a first-order sentence $X \supset Y$?
- Give a *sequent calculus proof* of the sentence

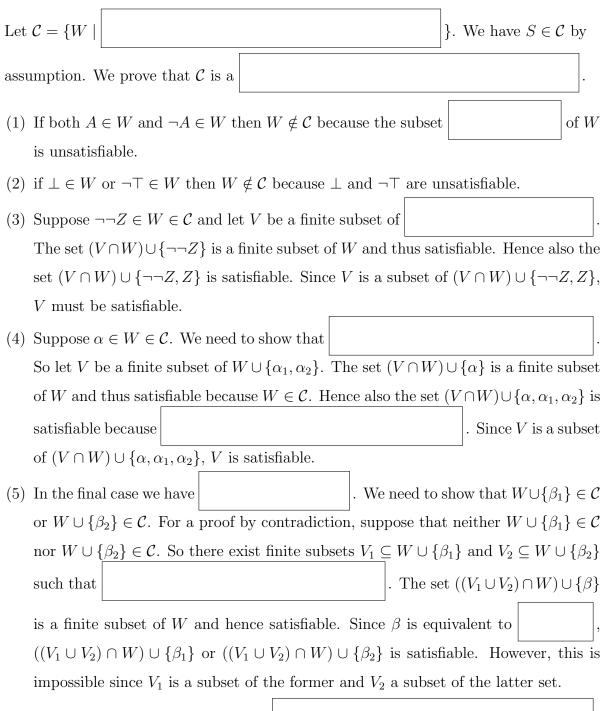
$$(\forall x)[P(x)\supset Q(x)]\supset [(\forall x)P(x)\supset (\forall x)Q(x)]$$

- 4 This exercise is about the propositional compactness theorem.
 - (a) State the propositional compactness theorem.

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(b) Complete the following proof of the propositional compactness theorem by filling in the missing parts.



The proof is concluded by an appeal to

5 This exercise is about Herbrand's theorem.

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- (a) Compute the Herbrand universe of the sentence $(\exists x)[R(f(x), a) \supset \neg(\exists y)R(b, f(y))].$
 - (b) Define the Herbrand expansion $\mathcal{E}(X, D)$ of an arbitrary sentence X over the domain $D = \{t_1, t_2\}.$
- 10 (c) Compute a tautologous Herbrand expansion for the valid sentence

$$(\forall z)(\exists w)(\forall x)[(\forall y)R(x,y) \supset R(w,z)]$$

10 6 Determine whether the following statements are true or false. Every correct answer is worth 2 points. For every wrong answer 1 point is subtracted, provided the total number of points is non-negative.

truefalsestatement $[\neg X \supset \bot\} \vdash_{ph} X$ $[\square]$ The set of all satisfiable propositional formulas is a Hintikka set. $[\square]$ The rank of the formula $(\exists x)(\forall y)[R(x, y) \supset \neg(\exists z)R(z, f(y))]$ is 5. $[\square]$ The sequent $X \supset Y, X \land Y \rightarrow \neg \neg X$ is an associated sequent of the set $\{X \land Y, \neg X, X \supset Y\}$. $[\square]$ The propositional formula $A \land (B \lor C)$ is an interpolant of the tautology $[A \land ((B \land D) \lor \neg C)] \supset \neg[(A \lor E) \supset \neg(C \supset B)]$.