Program Verification SS 2021 LVA 703083+703084

Sheet 9 Deadline: May 18, 2021, 8am

- Prepare your solutions on paper.
- Marking an exercise in OLAT means that a significant part of that exercise has been treated.
- Upload your Haskell files and your paper solution in OLAT, the latter as one PDF.

Exercise 1 *Implementation of Pattern Completeness* 4 p.

The task is to implement a function that tests a program on pattern completeness. In the negative case a term should be returned, whose constructor ground instances are not matched by any rule.

1. Implement a function to compute P_{init}. (2 points)
2. Implement the decision procedure, i.e., evaluation via $\rightharpoonup$. Note that the template file already contains a slightly modified matching algorithm from the last exercise sheet, which returns detailed error informations. (2 points)

Exercise 2 *Innermost Evaluation and Subterm Relation* 7 p.

1. The soundness proof of the subterm criterion uses the fact that $s \rhd t$ and $s \in NF(\hookrightarrow)$ implies $t \in NF(\hookrightarrow)$. Prove this fact. (2 points)
2. In the definition of $\overset{i}{\hookrightarrow}$ there occurs the condition $NF(\hookrightarrow)$, and not $NF(\overset{i}{\hookrightarrow})$. Show that this makes no difference by proving $NF(\overset{i}{\hookrightarrow}) = NF(\hookrightarrow)$. You can assume the obvious property $\overset{i}{\hookrightarrow} \subseteq \hookrightarrow$. (3 points)
3. Can you figure out why in the $\overset{i}{\hookrightarrow}$-definition, $NF(\hookrightarrow)$ was used instead of $NF(\overset{i}{\hookrightarrow})$?
   Hint: think about restrictions on inductively defined sets. (2 points)

Exercise 3 *Termination Analysis on Paper* 4 p.

Write your favourite sorting algorithm as functional program and try to prove termination via the subterm criterion. If the proof is not completed, indicate which dependency pairs remain.
Of course, here you also have to define a function for comparing natural numbers and other auxiliary functions. But you can assume that there are already datatypes Nat, List and Bool.

Exercise 4 *Termination Analysis in Haskell* 5 p.

1. Implement an algorithm to compute the set of dependency pairs of a functional program. (2 points)
2. Implement an algorithm that applies the subterm criterion if possible. (3 points)