

# Metamath

# What is it?

A Language

A Proof Explorer: [us.metamath.org](http://us.metamath.org)

A Proof Assistant: mmj2

Why is it cool?

# Only one rule: Substitution

Simple to implement: 300 LOC + 19 verifiers

Confidence: 5 implementations in 5 language by 5 people

Fast: verifies whole codebase 40.000 theorems in a few seconds.

Hyperlinked references: easy to follow

# Other Features

Proof checking separate from authoring

-> Small kernel -> Few Bugs + Fast

-> Big proof assistant (mmj2) w/ many features

74/100 of the Formalizing Theorems Challenge

Linear time checking

Demo

## Assertion

Ref	Expression
<b>idALT</b>	$\vdash (\varphi \rightarrow \varphi)$

### Proof of Theorem **idALT**

Step	Hyp	Ref	Expression
1		<a href="#">ax-1</a> 6	$\vdash (\varphi \rightarrow (\varphi \rightarrow \varphi))$
2		<a href="#">ax-1</a> 6	$\vdash (\varphi \rightarrow ((\varphi \rightarrow \varphi) \rightarrow \varphi))$
3		<a href="#">ax-2</a> 7	$\vdash ((\varphi \rightarrow ((\varphi \rightarrow \varphi) \rightarrow \varphi)) \rightarrow ((\varphi \rightarrow (\varphi \rightarrow \varphi)) \rightarrow (\varphi \rightarrow \varphi)))$
4	<a href="#">2</a> , <a href="#">3</a>	<a href="#">ax-mp</a> 5	$\vdash ((\varphi \rightarrow (\varphi \rightarrow \varphi)) \rightarrow (\varphi \rightarrow \varphi))$
5	<a href="#">1</a> , <a href="#">4</a>	<a href="#">ax-mp</a> 5	$\vdash (\varphi \rightarrow \varphi)$

Colors of variables: **wff** **setvar** **class**

Syntax hints:  $\rightarrow$  [wi](#) 4

This theorem was proved from axioms: [ax-mp](#) 5 [ax-1](#) 6 [ax-2](#) 7

This theorem is referenced by: [id1](#) 27643

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```
$( <MM> <PROOF_ASST> THEOREM=2p2e4 LOC_AFTER=9m1e8
```

```
df4::df-4      |- 4 = ( 3 + 1 )
```

```
df2::df-2      |- 2 = ( 1 + 1 )
```

```
df3::df-3      |- 3 = ( 2 + 1 )
```

```
1:df3:oveq1i  |- ( 3 + 1 ) = ( ( 2 + 1 ) + 1 )
```

```
2:df2:oveq2i  |- ( 2 + 2 ) = ( 2 + ( 1 + 1 ) )
```

```
d1::2cn       |- 2 e. CC
```

```
d2::ax-1cn    |- 1 e. CC
```

```
3:d1,d2,d2:addassi |- ( ( 2 + 1 ) + 1 ) = ( 2 + ( 1 + 1 ) )
```

```
4:3,1,2:3eqtr4ri |- ( 2 + 2 ) = ( 3 + 1 )
```

```
!qed:4,df4:eqtr4i  |- ( 2 + 2 ) = 4
```

```
$= ( c2 caddc co c3 c1 c4 2cn ax-1cn addassi df-3 oveq1i df-2 oveq2i
    3eqtr4ri df-4 eqtr4i ) AABCZDEBCZFAEBCZEBCAEIEBCZBCRQAEEGHHIDSEBJK
    ATABLMNOP $.
```

```
$)
```



# References

[us.metamath.org](http://us.metamath.org)

David A. Wheeler Metamath Tutorials

mmj2: built in tutorial

Lessons from Metamath by Mario Carneiro

# Proof steps

Unique step id or hypothesis or “qed”

Dependencies to previous steps

Reference to axiom or theorem justifying step

StepId:Dependencies:Reference      |- step

# Things that live in metamath

Well formed formulas: **greek letters**

Set variables: **lower case letters**

Class variables: **upper case letters**