universität innsbruck

Program Verification	SS 2023	LVA 703083+703084
Sheet 7		Deadline: May 10, 2023, 10am

- Prepare your solutions on paper.
- Mark the exercises in OLAT before the deadline.
- Marking an exercise means that a significant part of that exercise has been treated.

Exercise 1 *Lexicographic Combinations* **4 p.**

On slide 4/40, it was argued that termination holds because of a lexicographic measure. In this exercise we want to be a bit more formal about this aspect by showing that taking lexicographic combinations is a valid technique for termination proving.
In detail: A binary relation $\succ$ over some set A is strongly normalizing, if and only if there does not exist an infinite sequence of the form

$$a_0 \succ a_1 \succ a_2 \succ a_3 \succ \dots.$$

Given n binary relations $\succ_1, \dots, \succ_n$ over sets $A_1, \dots, A_n$, we define their lexicographic combination $\succ_{lex}$ as a binary relation over $A_1 \times \dots \times A_n$ as follows: a lexicographic decrease happens, if for some position i, the element at position i decreases w.r.t. $\succ_i$, the elements before position i are unchanged, and there is no restriction on the elements after position i. This can be made formal via the following inference rule:

$$\frac{1 \leq i \leq n \quad a_i \succ_i b_i}{(a_1, \dots, a_{i-1}, a_i, \dots, a_n) \succ_{lex} (a_1, \dots, a_{i-1}, b_i, \dots, b_n)}$$

Prove that whenever all $\succ_i$ are strongly normalizing for $1 \leq i \leq n$, then so is $\succ_{lex}$.

Exercise 2 *Pattern Completeness* **16 p.**

1. Consider the example execution on slide 4/46. For each of the pattern problems compute the multiset as described on slide 4/52, and observe the decrease. (3 points)
2. Prove that the multiset decreases for a split step. (5 points)
3. Consider the example on slide 4/45 and run this example using the improved algorithm of slide 4/53, i.e., via $\rightharpoonup'$. You may abbreviate *conj*, *True* and *False* by c, T and F, respectively. (4 points)
4. Formulate a refinement relation and specify a formal property that corresponds to "$\rightharpoonup'$ implements $\rightharpoonup$."
   Hint: You can write $\Rightarrow$ for performing a single step of a modified matching algorithm for linear terms. It consists of the decomposition step and also has a step $\{(x, t)\} \cup P \Rightarrow P$ to indicate that a variable always matches, cf. slide 3/23. (4 points)