Program Verification SS 2023 LVA 703083+703084

Sheet 8 Deadline: May 17, 2023, 10am

- Prepare your solutions on paper.
- Mark the exercises in OLAT before the deadline.
- Marking an exercise means that a significant part of that exercise has been treated.

Exercise 1 *Innermost Evaluation and Subterm Relation* 5 p.

1. The soundness proof of the subterm criterion uses the fact that $s \rhd t$ and $s \in NF(\hookrightarrow)$ implies $t \in NF(\hookrightarrow)$. Prove this fact. (2 points)
2. In the definition of $\overset{i}{\hookrightarrow}$ there occurs the condition $NF(\hookrightarrow)$, and not $NF(\overset{i}{\hookrightarrow})$. Show that this makes no difference by proving $NF(\overset{i}{\hookrightarrow}) = NF(\hookrightarrow)$. You can assume the obvious property $\overset{i}{\hookrightarrow} \subseteq \hookrightarrow$. (3 points)

Exercise 2 *Termination Analysis on Paper* 4 p.

Write your favourite sorting algorithm as functional program and try to prove termination via the subterm criterion and size-change termination. If the proof is not completed, indicate which dependency pairs remain. Of course, here you also have to define a function for comparing natural numbers and other auxiliary functions. But you can assume that there are already datatypes Nat, List and Bool.

Exercise 3 *Size-Change Termination* 11 p.

In the lecture the set of multigraphs $\mathcal{M}$ of a set of size-change graphs $\mathcal{G}$ has essentially been defined as follows:

$$\frac{G \in \mathcal{G}}{G \in \mathcal{M}} \qquad \frac{G_1 \in \mathcal{M} \quad G_2 \in \mathcal{M}}{G_1 \bullet G_2 \in \mathcal{M}}$$

Now consider the following set of multigraphs $\mathcal{N}$, defined as:

$$\frac{G \in \mathcal{G}}{G \in \mathcal{N}} \qquad \frac{G_1 \in \mathcal{G} \quad G_2 \in \mathcal{N}}{G_1 \bullet G_2 \in \mathcal{N}}$$

In this exercise we will show that both definitions are equivalent and also compare the subterm criterion with size-change termination.

1. Think about the relation-ship between the subterm criterion and size-change termination. Does one criterion subsume the other one? For both directions, either give a proof of the subsumption or provide a concrete counter example. (3 points)
2. Prove $\mathcal{N} \subseteq \mathcal{M}$. (2 points)
3. Prove $\mathcal{M} \subseteq \mathcal{N}$. You can assume that $\bullet$ is associative. Most likely, you will need to prove one auxiliary property. (4 points)
4. Think about an implementation: would it be faster to compute $\mathcal{M}$ or $\mathcal{N}$? Why? (2 points)