Homework

1. Apply the Branch & Bound algorithm to the formula (2 P)

$$\varphi = (q \vee \neg r) \wedge (p \vee q \vee r) \wedge \neg p \wedge (\neg p \vee \neg q) \wedge (p \vee q) \wedge (p \vee r) \wedge (\neg p \vee \neg q \vee r) \wedge \neg r$$

to determine $\mathrm{maxSAT}(\varphi)$.

2. On slide 18 we use a CNF encoding of a *cardinality constraint*.
   (a) Provide a concrete CNF φ_k^n encoding the cardinality constraint $CNF(x_1 + x_2 + \cdots + x_n \leq k)$ for arbitrary $n, k \in \mathbb{N}$. The size of the encoding should be polynomial in k and n. (2 P)
   (b) What is the space complexity (in k and n) of your encoding in big $\mathcal{O}$ notation? Could you improve on your complexity? (1 P)

3. Recall the variations of MaxSAT shown on slide 13.
   (a) Adapt the binary search procedure to also allow for *hard* and *soft* clauses. In other words, we want to find the maximum number of soft clauses that can be satisfied while all hard clauses must be satisfied. (2 P)
   (b) Further modify the procedure to maximize for the sum of weights of satisfied soft clauses, while satisfying all hard clauses. That is each soft clause C is associated with a weight $w(C)$, and we want to find the largest $\texttt{score} = \sum\{w(C) \mid C \text{ is a satisfied clause}\}$. (3 P)