Constraint Solving | SS 2024 | LVA 703305

Week 4 | April 12, 2024

Homework

1. Consider the formula

$$\varphi \equiv (x \neq v \vee v \neq y) \wedge (x = v \vee x \neq y) \wedge (x = y \vee x \neq z) \wedge z = w \wedge (x = z \vee w \neq y)$$

   Using DPLL(T) check the satisfiability of φ for the following theories.

   (a) T_1: The equality logic over the natural numbers where the single predicate $=$ is interpreted as the identity over $\mathbb{N}$. (2 P)

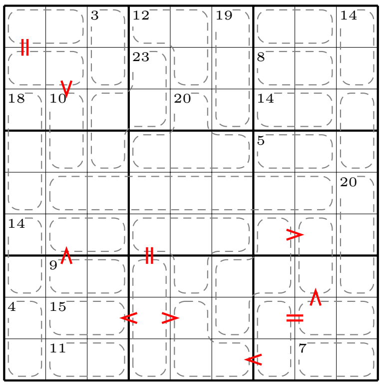
   (b) T_2: The equality logic over the Boolean values, where the single predicate $=$ is the identity over $\mathbb{B} = \{0, 1\}$. (2 P)

2. Consider the following instance γ of the Chinese Remainder Theorem: for all a and b there is an x such that $x \equiv a \pmod 7$ and $x \equiv b \pmod 5$. (2 P)

   (a) Can γ be expressed in Peano arithmetic? If yes, how?

   (b) Can γ be expressed in Presburger arithmetic? If yes, how?

3. Consider the following *Greater Than Killer Sudoku*. It follows the same rules as Killer Sudoku, but additionally adds equality ($=$) and greater than ($>$) constraints on the sum of some cages.



   (a) Encode this puzzle in Presburger arithmetic. (You are allowed to use $>$ as a predicate). (2 P)

   (b) Solve the puzzle using an SMT-solver (for example Z3). Is the solution unique? (2 P)

   Hint: To encode that some variables $x_1, \ldots, x_n$ are distinct you may just write $\mathit{distinct}(x_1, \ldots, x_n)$. SMT-LIB 2 also supports the constraint `(assert (distinct x1 x2 x3 ...))`.