universität innsbruck

Constraint Solving	SS 2024	LVA 703305
Week 5		April 19, 2024

Homework

1. Using the congruence closure algorithm for the sets of equations (2 P)

$$E_1 = \{\mathsf{g}(\mathsf{f}(x)) = \mathsf{f}(\mathsf{g}(x)),\ \mathsf{g}(\mathsf{f}(\mathsf{g}(y))) = x,\ \mathsf{g}(y) = x\}$$
$$E_2 = \{\mathsf{h}(\mathsf{h}(\mathsf{h}(\mathsf{a}))) = \mathsf{h}(\mathsf{a}),\ \mathsf{h}(\mathsf{h}(\mathsf{h}(\mathsf{h}(\mathsf{a})))) = \mathsf{a}\}$$

   determine which of the following entailments hold.

   (a) $E_1 \vDash \mathsf{f}(\mathsf{g}(x)) = x$ $\qquad$ $E_1 \vDash \mathsf{g}(\mathsf{f}(x)) = \mathsf{g}(x)$

   (b) $E_2 \vDash \mathsf{h}(\mathsf{h}(\mathsf{a}))) = \mathsf{a}$ $\qquad$ $E_2 \vDash \mathsf{h}(\mathsf{a}) = \mathsf{a}$

2. Based on the equality graph defined on slide 10 (3 P)

   (a) Construct an algorithm to find simple contradictory cycles. What is the complexity of your solution with respect to the number of edges in the graph?

   (b) Construct an algorithm to efficiently compute all possible T-propagate steps $C \models \ell$ for any literal ℓ for a given conjunction C of equalities and disequalities. What is the complexity of your solution with respect to the number of edges in the graph?

3. The congruence closure algorithm from slide 24 allows us to decide if $E \vDash_{\mathsf{EUF}} s = t$ for some set of ground equations E and ground equation $s = t$. However an EUF-solver should take a conjunction of ground equalities and disequalities and decide consistency.

   (a) Based on the congruence closure algorithm define an EUF-solver. (1 P)

   (b) We often want T-solvers to return a satisfying model if one exists. Define an algorithm which finds a model $\mathcal{M}$ given a satisfiable conjunctive EUF-formula φ such that $\mathcal{M} \vDash \varphi$. (2 P)

4. Let $F = \{C_1, \ldots, C_n\}$ be an unsatisfiable formula in clausal form. A subset $F' \subseteq F$ is called an unsatisfiable core, if it is itself unsatisfiable. There are many applications where it is interesting to find small unsatisfiable cores (e.g. see "Improvements"on slide 29 of week 4).

   Assume φ is an unsatisfiable formula from the conjunctive fragment of EUF. Design an algorithm to find an unsatisfiable core of φ. Explain how your algorithm works on the following formula (2 P)

$$\varphi \equiv \mathsf{a} = \mathsf{f}(\mathsf{a}) \wedge \mathsf{g}(\mathsf{a}) = \mathsf{f}(\mathsf{g}(\mathsf{a})) \wedge \mathsf{h}(\mathsf{f}(\mathsf{a})) = \mathsf{a} \wedge \mathsf{g}(\mathsf{f}(\mathsf{a})) \neq \mathsf{f}(\mathsf{g}(\mathsf{a}))$$

   Note: Your algorithm does not have to always find a minimal or smallest possible unsatisfiable core, but should also not just return the formula itself.