Homework

1. Consider the set I of linear inequalities over $\mathbb{Q}$:

$$y - 2x \leq 4 \qquad x - 2 \leq 3y \qquad x - 2y \geq 0 \qquad x + 2y \leq -2$$

   (a) Solve I using the simplex method, by hand on paper. Check that the solution you find is indeed a solution. (2 P)

   (b) Visualise I and the simplex solution process on it in a two-dimensional diagram (draw x, y within the range $[-3, \frac{1}{2}]$). (1 P)

2. Consider the following problem: (2 P)

   A chemical plant can produce n different products $a_1, \dots, a_n$. It can sell a product a_i for a price per liter of $\mathsf{price}(a_i)$ euros. All the products are made from m different raw ingredients $e_1, \dots, e_m$. The volume of ingredient e_i used to produce one liter of a product a_j is defined as $\mathsf{ing}(e_i, a_j)$. The company currently has $\mathsf{stock}(e_i)$ liters of ingredient e_i in stock. Is it possible for the company to produce at least R euros worth of products given its current stock of ingredients?

   (a) Assume that the $\mathsf{price}(a_i)$, $\mathsf{ing}(e_i, a_j)$ and $\mathsf{stock}(e_i)$ is known for all products and ingredients. Find a LRA formula φ which encodes the problem. Note that also parts of a product can be sold, e.g., half a liter.

   (b) Test your encoding using an SMT solver for the instance of the problem with three products a_1, a_2, a_3 and three ingredients e_1, e_2, e_3. The current stock of ingredients is $\mathsf{stock}(e_1) = 50$, $\mathsf{stock}(e_2) = 200$, $\mathsf{stock}(e_3) = 120$, and the prices of products as well as ingredient required to produce a product are

i	$\mathsf{price}(a_i)$	$\mathsf{ing}(e_1, a_i)$	$\mathsf{ing}(e_2, a_i)$	$\mathsf{ing}(e_3, a_i)$
1	160	0	5	2
2	200	1	2	4
3	210	4	0	3

   Can the plant produce products worth more than 9100 euros? How about 9200 euros?

3. Extend the algorithm for difference logic to arbitrary constraints, i.e., including strict inequalities, such as the following ones.

$$x - y < -3 \qquad x - w \leq -4 \qquad y - z < 5 \qquad z - w \leq -2 \qquad z - x \leq -1$$

   (a) Show how strict inequalities can be added for $\mathbb{Z}$ and test it on the example constraints. (1 P)

   (b) Show how strict inequalities can be added for $\mathbb{Q}$ and test it on the example constraints. (2 P)

4. In this exercise you will prove the lemma on slide 26. (2 P)
   Let C be a conjunction of LRA atoms including $\geq$ and $>$, and C_δ the LRA conjunction of LRA atoms without $>$, obtained by the transformation on slide 26.

   (a) If there exists a rational such $\delta > 0$ such that C_δ is satisfiable, then C is satisfiable.

   (b) If C is satisfiable, then there exists a rational number $\delta > 0$ such that C_δ is satisfiable.