universität innsbruck

Constraint Solving	SS 2024	LVA 703305
Week 9		May 17, 2024

Homework

1. Consider the following inequalities as part of a larger LIA problem. Which bounds can be tightened? What are the tighter bounds? (1 P)

   (i) $91x + 49y \leqslant 55$ (ii) $39x + 27y \leqslant 21$ (iii) $33x + 51y < 27$

2. Find the implied equations (in $\mathbb{Q}$) of the following inequalities. You may use the simplex implementation to find minimal unsatisfiable cores. (1 P)

   $$-2x + y \leqslant -2 \qquad x + 3y \leqslant 8$$
   $$6x - 3y \leqslant 7 \qquad x - 2y \leqslant -2$$

3. Prove the only-if direction of the "Cube Inclusion for Single Inequality" lemma on slide 11. In other words, prove that for all $\vec{a}, \vec{z} \in \mathbb{Q}^n$, $c \in \mathbb{Q}$, and $s \in \mathbb{Q}_{\geqslant 0}$ the following implication holds: (2 P)

   $$\mathsf{cube}_s(\vec{z}) \subseteq \{\vec{x} \mid \vec{a} \cdot \vec{x} \leqslant c\} \implies \vec{a} \cdot \vec{z} \leqslant c - s \sum_{i=1}^{n} |a_i|$$

4. Consider the Diophantine Equation Solver from slide 24.

   (a) Use the algorithm to check if an integer solution exists for the following Diophantine equations. If yes, what is the solved form returned by the algorithm? (2 P)

   $$6x + 10y + 26z = 14$$
   $$7x + 12y + 21z = 29$$

   (b) Sometimes we are not only interested in the solved form, but also an integer solution to the equations. Explain how we can obtain a solution to a set of Diophantine equations from a solved form. (1 P)

5. Prove that the Diophantine Equation Solver of Griggio (slide 24) terminates for all inputs E. (3 P)