universität innsbruck

Constraint Solving	SS 2024	LVA 703305
Week 12		June 7, 2024

Homework

1. Apply Ferrante and Rackoff's method in order to convert the following formula into an equivalent quantifier free formula. (2 P)

   $$\varphi := \forall y.\exists x.4y + 8 > 10x \wedge 2y < 6x + 3$$

   Remark: perform basic arithmetic simplifications after having eliminated y, and before eliminating x. However, perform the elimination of x also via Ferrante and Rackoff's method and do not use "obvious arithmetic reasoning" to immediately see the truth value of φ.

2. Prove soundness of the essential step of Ferrante and Rackoff's method, i.e., the equivalence between $\exists x.\varphi_3(x)$ and φ_4, cf. slide 9.

   Hint: Perform a similar argumentation as in the soundness proof of Cooper's method.

   (a) Show that whenever φ_4 is satisfiable then so is $\exists x.\varphi_3(x)$. (1 P)

   (b) Show that whenever $\exists x.\varphi_3(x)$ is satisfiable then so is φ_4. (2 P)

3. Consider the following formula for applying Cooper's method.

   $$\varphi := \forall x\, y\, z.10x - 15y + 7 \neq 45z$$

   (a) Convert φ into an equivalent formula ψ where the quantification of z is removed. Write down each step that is performed in Cooper's method. You can use intermediate arithmetic simplifications and should simplify the final formula. (1 P)

   (b) The formula that you have computed in the previous exercise might be:

   $$\psi := \neg\exists x\, y.45 \mid 10x - 15y + 7 \equiv \varphi$$

   Transform ψ further into an equivalent quantifier free formula χ using Cooper's method. Use the optimizations from the lecture! (1 P)

   (c) Finally compute whether χ is valid, e.g., by writing a computer program that evaluates χ. (1 P)

4. Both in Ferrante and Rackoff's method (slide 7) and in Cooper's method (slide 13) the formula φ is first transformed into NNF. In this exercise we consider if this is a necessary step.

   (a) Is Ferrante and Rackoff's method still sound if we do not transform φ to NNF, but directly start with step 2? Explain why it is, or give a counter example. (1 P)

   (b) Is Cooper's method still sound if we do not transform φ to NNF, but directly start with step 2? Explain why it is, or give a counter example. (1 P)