

Algorithmische Mathematik 7

Logic in Computer Science

This exam consists of four exercises. Please explain how you solved each exercise. The available points for each item are written in the margin. You need at least 50 points to pass.

1. For each of the following sequents, either give a proof or explain why a proof does not exist:

- [7] (a)  $p \rightarrow q, p \rightarrow \neg q \vdash \neg p$
- [7] (b)  $p \vee q, \neg p \rightarrow q \vdash q \rightarrow p$
- [7] (c)  $\neg(p \wedge q) \vdash \neg q \vee \neg p$

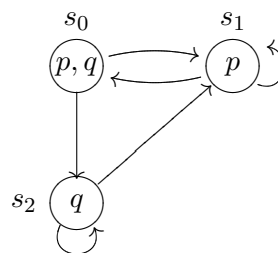
2. Consider the boolean formula  $f(x, y, z) = x \cdot (\bar{y} \oplus z) + y \cdot \bar{z}$ .

- [7] (a) Write down its truth table.
- [7] (b) Compute the unique reduced OBDD with respect to the ordering  $[x, y, z]$ .
- [7] (c) Compute the unique reduced OBDD with respect to the ordering  $[z, x, y]$ .
- [7] (d) Compute a reduced OBDD for  $\exists y.f$ .

3. For each of the following formulas of predicate logic, either give a proof or find a model which does not satisfy it:

- [8] (a)  $(\forall y P(a, y) \wedge \forall x \forall y (P(x, y) \rightarrow P(f(f(x)), y))) \rightarrow \forall x P(x, a)$
- [8] (b)  $\forall x \forall y (P(x) \rightarrow Q(y)) \rightarrow (\exists x P(x) \rightarrow \forall y Q(y))$
- [8] (c)  $(\forall x (P(x) \rightarrow Q(x)) \wedge P(a)) \rightarrow (a = b \rightarrow Q(b))$

4. Consider the following CTL model  $\mathcal{M}$ :



- [9] (a) Determine in which states the formula  $A[p \text{ U } \neg p \wedge q]$  holds.
- [9] (b) Determine in which states the formula  $EX(\neg AX(p \wedge EX q))$  holds.
- [9] (c) Find a formula  $\phi$  such that  $AF \phi$  is satisfied in state  $s_0$  but not in state  $s_1$ .