

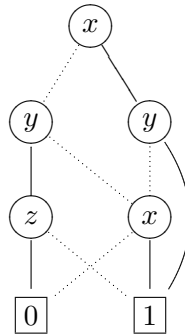
Algorithmische Mathematik 7

Logic in Computer Science

This exam consists of four exercises. Please explain how you solved each exercise. The available points for each item are written in the margin. You need at least 50 points to pass.

- [6] 1. (a) Give the definition of conjunctive normal form (CNF).
 [6] (b) Find an equivalent formula in CNF for the formula $\neg(p \wedge q) \wedge r \rightarrow (\neg p \rightarrow r \vee s)$
 [6] (c) Is the formula of part (b) valid?
 [6] (d) Is the Horn formula $(\top \rightarrow p) \wedge (p \rightarrow q) \wedge (p \wedge r \rightarrow q) \wedge (q \wedge r \rightarrow s) \wedge (s \rightarrow t) \wedge (t \rightarrow \perp)$ satisfiable?

2. Consider the following BDD B :

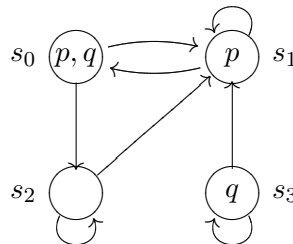


- [5] (a) Is B reduced?
 [5] (b) Is B ordered?
 [6] (c) Find an equivalent reduced OBDD with respect to the ordering $[x, y, z]$.
 [6] (d) Which boolean function does B represent?
 [6] (e) Compute $\text{restrict}(1, y, B)$.

3. For each of the following formulas of predicate logic, either give a proof or find a model which does not satisfy it:

- [8] (a) $(\forall y P(a, f(y)) \wedge \forall x \forall y (P(x, y) \rightarrow P(f(f(x)), f(y)))) \rightarrow \exists x P(x, x)$
 [8] (b) $(\forall x (P(x) \rightarrow Q(f(x))) \wedge P(a)) \rightarrow (\neg(a = b) \vee Q(f(b)))$
 [8] (c) $\forall x \exists y (P(x) \rightarrow Q(y)) \rightarrow (\exists x P(x) \rightarrow \forall y Q(y))$

4. Consider the following CTL model \mathcal{M} :



- [8] (a) Determine in which states the formula $E[AF q U \neg AX p]$ holds.
 [8] (b) Determine in which states the formula $AX(p \wedge EX(\neg AX(p \vee EX q)))$ holds.
 [8] (c) Find a formula ϕ such that $EG \phi$ is satisfied in state s_2 but not in state s_1 .