

Algorithmische Mathematik 7

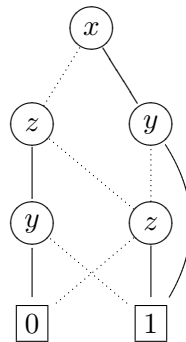
Logic in Computer Science

This exam consists of four exercises. *Explain how you solved each exercise.* The available points for each item are written in the margin. You need at least 50 points to pass.

1. For each of the following sequents, either give a proof or explain why a proof does not exist:

- [7] (a)  $\neg(p \rightarrow q) \vdash q \wedge \neg p$
- [7] (b)  $\neg p \vee q, p \rightarrow q \vdash \neg p$
- [7] (c)  $p \rightarrow q \vee r, q \rightarrow \neg p, \neg r \rightarrow p \vdash q \rightarrow r$

2. Consider the following BDD  $B$ :

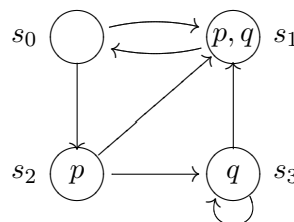


- [6] (a) Is  $B$  reduced?
- [6] (b) Is  $B$  ordered?
- [6] (c) Find an equivalent reduced OBDD with respect to the ordering  $[x, y, z]$ .
- [6] (d) Which boolean function does  $B$  represent?

3. For each of the following formulas of predicate logic, either give a proof or find a model which does not satisfy it:

- [8] (a)  $\forall x \exists y (P(x) \rightarrow Q(y)) \rightarrow \forall x P(x)$
- [8] (b)  $(\forall x (P(x) \rightarrow Q(f(x))) \wedge P(a)) \rightarrow (a = f(a) \rightarrow Q(f(f(a))))$
- [8] (c)  $(\exists y P(a, f(y)) \wedge \forall x \forall y (P(x, f(y)) \rightarrow P(f(x), y))) \rightarrow \exists x P(x, x)$

4. Consider the following CTL model  $\mathcal{M}$ :



- [8] (a) Determine in which states the formula  $E[\neg AX p \cup EX(p \wedge q)]$  holds.
- [8] (b) Find a formula  $\phi$  such that  $EG \phi$  is satisfied in state  $s_0$  but not in state  $s_2$ .
- [9] (c) Construct a reduced OBDD that represents the transition relation of  $\mathcal{M}$ .