University of Innsbruck — Institute of Computer Science
1st Exam — Wednesday, June 29, 2005, 11:30–13:30

Algorithmische Mathematik 7 — Logic in Computer Science

Name: **MatrNr:** **StudienKZ:**

This exam consists of four exercises. Explain how you solved each exercise. The available points for each item are written in the margin. You need at least 50 points to pass.

[7] 1. (a) Give a natural deduction proof of the sequent $\neg p \vdash p \to (p \to q)$.

[7] (b) Is the formula $(p \to \neg q) \to (q \vee \neg p)$ valid?

[7] (c) Is the formula $((\neg p \vee (\neg q \vee \neg r)) \wedge ((p \vee \neg q) \wedge ((p \vee (q \vee r)) \wedge ((q \vee \neg r) \wedge (r \vee \neg p)))))$ satisfiable?

2. Consider the boolean formula $f(x, y, z) = (x + y) \cdot (\overline{z} + x) + \overline{y} \cdot z$.

[7] (a) Write down its truth table.

[7] (b) Compute the unique reduced OBDD with respect to the ordering $[x, y, z]$.

[7] (c) Compute the unique reduced OBDD with respect to the ordering $[z, x, y]$.

[7] (d) Compute a reduced OBDD for $\forall z.f$.

[8] 3. (a) Complete the following proof of the sequent

$$\forall x\,(P(x) \vee Q) \vdash \forall x\,P(x) \vee Q$$

by filling in the missing parts:

1		$\forall x\,(P(x) \vee Q)$	premise
2		$Q \vee \neg Q$	LEM
3		Q	assumption
4			
5		$\neg Q$	assumption
6	x_0		
7		$P(x_0)$	assumption
8		Q	assumption
9			
10			
11		$P(x_0)$	$\vee$e 6, 7, 8-10
12		$\forall x\,P(x)$	$\forall$i 6–11
13		$\forall x\,P(x) \vee Q$	$\vee i_1$ 12
14		$\forall x\,P(x) \vee Q$	$\vee$e 2, 3–4, 5–13

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[8] (b) Consider the model $\mathcal{M} = (A, R^{\mathcal{M}})$ with $A = \{s_0, s_1, s_2\}$ and $R^{\mathcal{M}} = \{(s_0, s_1), (s_1, s_0), (s_1, s_1), (s_1, s_2), (s_2, s_0)\}$. Does the formula

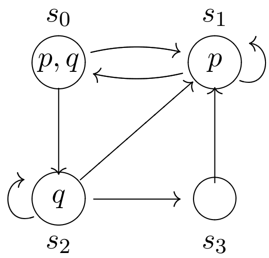
$$\forall x\, (\neg R(x,x) \to \exists y\, (R(x,y) \wedge \neg R(y,x)))$$

hold in $\mathcal{M}$?

[8] (c) Repeat part (b) for the second-order formula

$$\forall P\, (\forall x\, \neg P(x,x) \vee \exists x\, \forall y\, (R(x,y) \to P(x,y)))$$

[9] 4. (a) Determine in which states of the following model the CTL formula $\mathsf{E}[p\,\mathsf{U}(\neg p \wedge \mathsf{EX}\,q)]$ holds:



[9] (b) Are the LTL formulas $\mathsf{F}(p \wedge q)$ and $\mathsf{F}\,p \wedge \mathsf{F}\,q$ equivalent?

[9] (c) Are the CTL* formulas $\mathsf{F}\,\mathsf{G}\,p$ and $\mathsf{AF}\,\mathsf{AG}\,p$ equivalent?