UNIVERSITY OF INNSBRUCK | INSTITUTE OF COMPUTER SCIENCE
2ND EXAM | FRIDAY, SEPTEMBER 30, 2005, 9:00–11:00

Algorithmische Mathematik 7 — Logic in Computer Science

Name: **MatrNr:** **StudienKZ:**

This exam consists of four exercises. Explain how you solved each exercise. The available points for each item are written in the margin. You need at least 50 points to pass.

[6] 1. (a) Find an equivalent formula in conjunctive normal form for the formula

$$(p \wedge q) \wedge \neg r \rightarrow ((r \vee q) \rightarrow \neg p)$$

[6] (b) Is the formula of part (a) satisfiable?

[6] (c) Is the formula of part (a) valid?

[6] (d) Is the Horn formula

$$(\top \rightarrow p) \wedge (q \rightarrow r) \wedge (\top \rightarrow q) \wedge (p \wedge s \rightarrow t) \wedge (t \rightarrow s) \wedge (q \wedge t \rightarrow \bot)$$

satisfiable?

2. Consider the boolean formula $f(x, y, z) = (x + \overline{y}) \cdot (\overline{z} + x) \cdot (\overline{y} + z)$.

[7] (a) Write down its truth table.

[7] (b) Compute the unique reduced OBDD with respect to the ordering $[x, y, z]$.

[7] (c) Compute the unique reduced OBDD with respect to the ordering $[y, z, x]$.

[7] (d) Compute a reduced OBDD for $\exists x.f$.

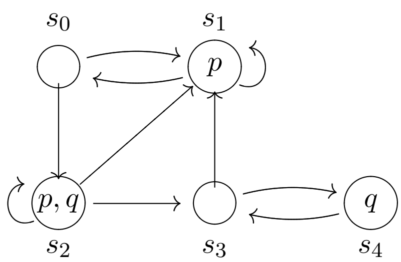
3. For each of the following formulas of predicate logic, either give a proof or find a model which does not satisfy it:

[8] (a) $\forall x\, \exists y\, (P(x, y) \rightarrow P(y, x)) \rightarrow \exists x\, P(x, x)$

[8] (b) $\forall x\, (P(f(x)) \rightarrow \neg Q(x)) \wedge Q(a) \rightarrow (P(f(b)) \rightarrow \neg(a = b))$

[8] (c) $\exists x\, \forall y\, \forall z\, (P(x, y) \rightarrow P(x, z)) \rightarrow (\neg(\forall x\, \exists y\, P(x, y)) \vee \forall x\, \forall y\, P(x, y))$

[8] 4. (a) Determine in which states of the model



the CTL formula $\mathsf{A}[\mathsf{A}[p \,\mathsf{U}\, q] \,\mathsf{U}\, (\mathsf{EX}\, q \wedge \mathsf{A}[q \,\mathsf{U}\, \neg p])]$ holds.

[8] (b) Are the LTL formulas $\mathsf{G}(p \vee q)$ and $\mathsf{G}\, p \vee \mathsf{G}\, q$ equivalent?

[8] (c) Are the CTL* formulas $\mathsf{X}\,\mathsf{F}\, p$ and $\mathsf{AF}\,\mathsf{AX}\, p$ equivalent?