Algorithmische Mathematik 7 — Logic in Computer Science
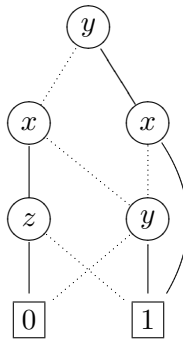
Name: **MatrNr:** **StudienKZ:**

This exam consists of four exercises. Explain how you solved each exercise. The available points for each item are written in the margin. You need at least 50 points to pass.

1. For each of the following sequents, either give a proof or explain why a proof does not exist:

[7]   (a) $p \to q, p \vee \neg q \vdash \neg p$

[7]   (b) $\neg(p \to q) \vdash \neg p \wedge q$

[7]   (c) $p \to q \vee r, r \to \neg p, \neg q \to p \vdash r \to q$

2. Consider the following BDD B:

[5]   (a) Is B reduced?

[5]   (b) Is B ordered?

[6]   (c) Find an equivalent reduced OBDD with respect to the ordering $[x, y, z]$.

[6]   (d) Which boolean function does B represent?

[6]   (e) Compute `restrict`$(0, z, B)$.

3. For each of the following formulas of predicate logic, either give a proof or find a model which does not satisfy it:

[8]   (a) $(\forall x\, P(x, a) \wedge \forall x\, \forall y\, (P(x, y) \to P(x, f(f(y))))) \to \forall y\, P(a, y)$

[8]   (b) $\exists x\, \forall y\, \forall z\, (P(x, y) \to P(x, z)) \to (\neg(\forall x\, \exists y\, P(x, y)) \vee \forall x\, \forall y\, P(x, y))$

[8]   (c) $(\forall x\, (P(x) \to Q(x)) \wedge P(a)) \to (a = b \to Q(b))$

4. Consider the following model $\mathcal{M}$:
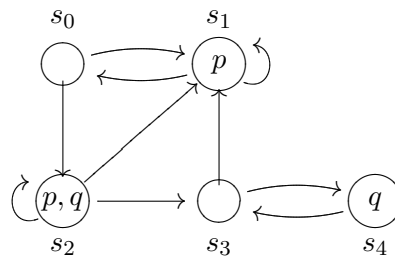
[9]   (a) Determine in which states the CTL formula $\mathsf{E}[\mathsf{AF}\, q \,\mathsf{U}\, \neg\, \mathsf{AX}\, p]$ holds.

[9]   (b) Determine in which states the CTL formula $\mathsf{AX}(p \wedge \mathsf{EX}(\neg\, \mathsf{AX}(p \vee \mathsf{EX}\, q)))$ holds.

[9]   (c) Find a CTL formula ϕ such that $\mathsf{EG}\, \phi$ is satisfied in state s_0 but not in state s_3.