Algorithmische Mathematik 7 — Logic in Computer Science

This exam consists of four exercises. *Explain your answers.* The available points for each item are written in the margin. You need at least 50 points to pass.

1 Consider the following questions concerning propositional logic.

[8] (a) Give a natural deduction proof of the sequent $p \vee q, \neg p \vee \neg q \vdash (p \wedge \neg q) \vee (\neg p \wedge q)$.

[7] (b) Is the formula $\neg((p \wedge q) \vee (r \wedge s))$ satisfiable?

[7] (c) Test the satisfiability of the formula in (b) with the linear SAT solver.

[7] (d) Test the satisfiability of the formula in (b) with the cubic SAT solver.

2 Consider the boolean function $f(x, y, z) = (x \cdot y \cdot z) \oplus (x + y + z)$.

[7] (a) Write down a truth table for f.

[7] (b) Construct a reduced OBDD for f with respect to the variable ordering $[x, y, z]$.

[7] (c) Construct a reduced OBDD for $\exists y.f$ by using apply and restrict. Give all intermediate OBDDs.

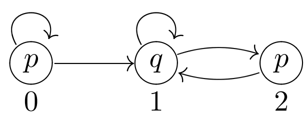
3 Determine whether the following formulas of predicate logic are valid and/or satisfiable. Give natural deduction proofs for the valid ones.

[7] (a) $\phi_1 = \forall x\,(P(x) \vee Q(x)) \rightarrow \exists x\,Q(x)$

[7] (b) $\phi_2 = \forall x\,(P(x) \vee \neg P(x)) \rightarrow \exists x\,(P(x) \wedge \neg P(x))$

[7] (c) $\phi_3 = \exists x\,\exists y\,(P(x, y) \vee P(y, x)) \rightarrow \exists x\,\exists y\,P(x, y)$

4 Consider the model $\mathcal{M}$:



[7] (a) Determine in which states of $\mathcal{M}$ the CTL formula $\mathsf{E}[\mathsf{EF}\,q\,\mathsf{U}\,\mathsf{AF}\,p]$ holds.

[7] (b) Determine in which states of $\mathcal{M}$ the LTL formula $\mathsf{F}\,q\,\mathsf{U}\,\mathsf{F}\,p$ holds.

[7] (c) Give a model which shows that $\mathsf{AF}\,\mathsf{EF}\,p$ and $\mathsf{AF}\,p$ are not equivalent.

[8] (d) Give an LTL formula ϕ that holds in state 0 but not in states 1 and 2 of $\mathcal{M}$.