University of Innsbruck — Institute of Computer Science
2nd Exam — Friday, September 29, 2006, 09:00–11:00

Algorithmische Mathematik 7 — Logic in Computer Science

This exam consists of four exercises. *Explain your answers.* The available points for each item are written in the margin. You need at least 50 points to pass.

1 Consider the following questions concerning propositional logic.

[8] (a) Give a natural deduction proof of the sequent $\neg(p \wedge q) \vdash \neg p \vee \neg q$.

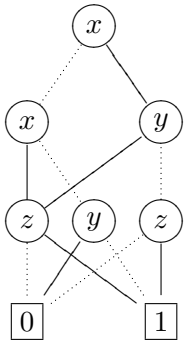
[8] (b) Consider the formula $p \to (\neg q \to p)$. Find an equivalent conjunctive normal form.

[8] (c) Is the Horn formula

$$(\top \to p) \wedge (q \to r) \wedge (\top \to q) \wedge (p \wedge r \to t) \wedge (q \wedge t \to s) \wedge (s \wedge r \to \bot)$$

satisfiable?

2 Consider the following BDD B:



[6] (a) Is B reduced?

[6] (b) Is B ordered?

[6] (c) Construct an equivalent reduced OBDD with respect to the variable ordering $[z, x, y]$.

[6] (d) Which boolean function does B represent?

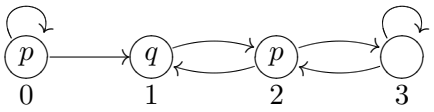
3 Determine whether the following formulas of predicate logic are valid and/or satisfiable. Give natural deduction proofs for the valid ones.

[9] (a) $\phi_1 = (\forall x\, \exists y\, (P(x) \to Q(y))) \to \forall x\, (P(x) \to \exists y\, Q(y))$

[9] (b) $\phi_2 = (\forall x\, \exists y\, (P(y) \to Q(x))) \to \forall x\, (\exists y\, P(y) \to Q(x))$

[9] (c) $\phi_3 = (\exists x\, \forall y\, (P(x) \to Q(y))) \to \exists x\, (P(x) \to \forall y\, Q(y))$

4 Consider the model $\mathcal{M}$:



[8] (a) Determine in which states of $\mathcal{M}$ the CTL formula $\mathsf{A}[\mathsf{A}[p\,\mathsf{U}\,q]\,\mathsf{U}(\mathsf{EX}\,q \vee \mathsf{E}[q\,\mathsf{U}\,p])]$ holds.

[8] (b) Determine in which states of $\mathcal{M}$ the LTL formula $\mathsf{F}\,q\,\mathsf{U}\,\mathsf{X}\,p$ holds.

[8] (c) Give an LTL formula ϕ that holds in states 0 and 3 but not in states 1 and 2 of $\mathcal{M}$.