# Algorithmische Mathematik 7 — Logic in Computer Science

This exam consists of four exercises. ***Explain your answers.*** The available points for each item are written in the margin. You need at least 50 points to pass.

**1** Consider the propositional formula $\phi = p \land q \land r \to q$.

[8] (a) Is $\phi$ a Horn formula? If yes, use the marking algorithm to check the satisfiability of $\phi$. If no, explain why $\phi$ is not a Horn formula.

[8] (b) Transform the *negation* of $\phi$ into an equivalent conjunctive normal form.

[9] (c) Use resolution on the result of (b) to check the satisfiability of $\neg\phi$. What does the outcome mean for $\phi$?

**2** Consider the boolean function

$$f(x, y, z) = \begin{cases} 0 & \text{if the majority of } x, y \text{ and } z \text{ are } 1 \\ 1 & \text{otherwise} \end{cases}$$

[8] (a) Give a binary decision tree for $f$ with the variable ordering $[z, y, x]$ and use the reduce algorithm to construct an equivalent reduced OBDD.

[8] (b) Use Shannon's expansion to obtain a boolean expression that is equivalent to $f$.

[8] (c) Construct a reduced OBDD for $\forall z.f$ by using apply and restrict. Give all intermediate OBDDs.

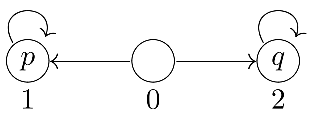
**3** Determine whether the following formulas of predicate logic are valid and/or satisfiable. Give natural deduction proofs for the valid ones.

[9] (a) $\phi_1 = \forall x\, \forall y\, (P(y) \to Q(x)) \to \exists y\, P(y) \to \forall x\, Q(x)$

[9] (b) $\phi_2 = \forall x\, \exists y\, (P(x) \to Q(y)) \to \forall x\, (P(x) \to \exists y\, Q(y))$

[9] (c) $\phi_3 = \exists x\, \forall y\, (P(x) \to Q(y)) \to \exists y\, P(y) \to \forall x\, Q(x)$

**4** Consider the model $\mathcal{M}$:




and the CTL formula $\phi = \mathsf{EF}\, p \land \mathsf{EF}\, q$.

[8] (a) Determine in which states of $\mathcal{M}$ the formula $\phi$ holds.

[8] (b) Is the CTL formula $\psi = \mathsf{EF}(p \land q)$ equivalent to $\phi$?

[8] (c) Give an LTL formula that is equivalent to the negation of $\phi$.