universität innsbruck

Logik | SS 2020 | LVA 703027

EXAM 1 | June 22, 2020

This exam consists of five exercises. The available points for each item are written in the margin. You need at least 50 points to pass. ***Explain your answers!!***

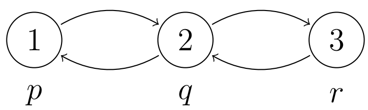
1 Consider the boolean functions $f(x, y, z) = (x + y) \oplus (y + z)$ and $g(x, y) = \overline{x} \oplus y$.

[6] (a) Is f monotone? Is f self-dual? Is f affine?

[7] (b) Compute a reduced OBDD for $\overline{f(x, y, z)} + g(x, y)$ with variable ordering $[x, y, z]$.

[7] (c) Can $g(x, y)$ be expressed (only) using f and the variables x and y?

2 Consider the model $\mathcal{M}$:



[7] (a) Use the CTL model checking algorithm to determine in which states of $\mathcal{M}$ the CTL formula $\varphi = \mathsf{AG}(\mathsf{E}[\mathsf{EF}\, p\, \mathsf{U}\, r])$ holds.

[6] (b) Consider the LTL formulas $\psi_1 = \mathsf{G}\, p$, $\psi_2 = \mathsf{F}\, q$, and $\psi_3 = \mathsf{X}\, r$. For all $1 \leqslant i, j \leqslant 3$ determine whether $\mathcal{M}, i \vDash \psi_j$ holds or not.

[7] (c) Show that the CTL formulas $\chi_1 = p \to \mathsf{AG}\, q$ and $\chi_2 = \mathsf{AG}(p \to q)$ are not equivalent.

3 For each of the following sequents, either give a natural deduction proof or find a model which does not satisfy it.

[6] (a) $p \vee \neg q,\ \neg p \to q,\ p \to r \ \vdash\ p \wedge r$

[7] (b) $\forall x\, \forall y\, (Q(x, y) \to x = y),\ \exists x\, \exists y\, Q(x, y) \ \vdash\ \exists x\, \forall y\, Q(x, y)$

[7] (c) $\forall x\, \forall y\, (x = y \to \neg P(x, y)),\ \forall x\, (Q(x) \to P(x, x)) \ \vdash\ \forall x\, \neg Q(x)$

[7] $\boxed{4}$ (a) Use resolution to determine whether the formula

$$\varphi = (r \to p \vee q) \wedge (p \vee q \vee r) \wedge \neg((q \to p) \to p)$$

is satisfiable or not.

[6] (b) Use DPLL to determine satisfiability of the CNF

$$\psi = (p \vee q \vee r) \wedge (p \vee \neg q \vee r) \wedge (q \vee \neg r) \wedge (\neg q \vee \neg r) \wedge (\neg p \vee s) \wedge (\neg p \vee r \vee \neg s)$$

[7] (c) Determine whether the terms $f(x, h(x), z, g(a))$ and $f(g(z), h(g(y)), y, x)$ are unifiable and compute a most general unifier if possible. Here x, y, z are variables and a is a constant.

[20] $\boxed{5}$ Determine whether the following statements are true or false. Every correct answer is worth 2 points. For every wrong answer 1 point is subtracted, provided the total number of points is non-negative.

statement
The LTL formulas $\mathsf{FGF}p$ and $\mathsf{GF}p$ are equivalent.
Every prenex normal form is a Skolem normal form.
The boolean function $f(x, y) = x \oplus y \oplus x$ is monotone.
The proof rule $\bot$e is a derived rule in natural deduction.
The boolean function $f(x, y, z) = x \oplus (y + \overline{z})$ is self-dual.
The terms $f(x, x, y, g(y))$ and $f(y, g(z), g(z), x)$ are unifiable.
The clause $\{p, \neg p\}$ is a resolvent of the clauses $\{\neg p, q\}$ and $\{p, \neg q\}$.
It is decidable whether a set of predicate logic formulas is consistent.
The sequent $\vdash \forall x\, \exists y\, (P(x) \to Q(y)) \to \forall x\, \neg(P(x) \wedge \exists y\, \neg Q(y))$ is valid.
If $f(x, y)$ is an adequate boolean function then also $g(x, y, z) = f(x, y) + z$ is adequate.