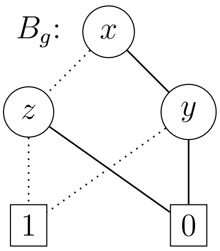
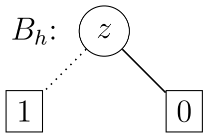
Logik | SS 2020 | LVA 703027

EXAM 2 — September 25, 2020

This exam consists of <u>five</u> exercises. The available points for each item are written in the margin. You need at least 50 points to pass.

1 Consider the boolean function f defined by $f(x, y, z) = x\overline{y} \oplus y\overline{z}$ and the following two reduced OBDDs:



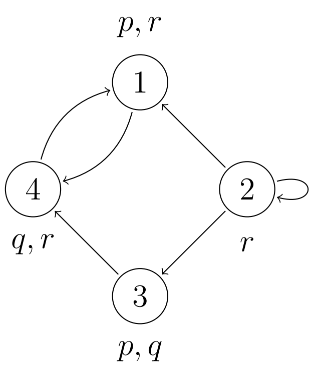


[6] (a) Compute a reduced OBDD for $f(x, y, z)$ with the variable ordering $[x, y, z]$.

[7] (b) Compute $\mathsf{apply}(+, B_g, B_h)$.

[7] (c) Show that the set $\{f, \rightarrow\}$ is adequate.

2 Consider the model $\mathcal{M}$:



[7] (a) Use the CTL model checking algorithm to determine in which states of $\mathcal{M}$ the CTL formula $\phi = \mathsf{EG}(r \wedge \mathsf{AF}\, \mathsf{A}[q\, \mathsf{U}\, p])$ holds.

[6] (b) Consider the LTL formula $\psi = \mathsf{G}\, \mathsf{F}\, p \rightarrow \mathsf{F}(q \wedge r)$. For each state $1 \leqslant i \leqslant 4$ determine whether $\mathcal{M}, i \vDash \psi$ holds or not.

[7] (c) Consider the CTL formula $\chi = \mathsf{EF}\, p$. Either give an LTL formula χ' which is equivalent to χ or explain why χ is not expressible in LTL.

3 For each of the following sequents, either give a natural deduction proof or find a model which does not satisfy it.

[6] (a) $\vdash\ \neg p \vee q \rightarrow (p \rightarrow q)$

[6] (b) $\vdash\ ((p \rightarrow q) \rightarrow q) \rightarrow ((q \rightarrow p) \rightarrow p)$

[8] (c) $\forall x\, (P(x) \vee Q(x)),\ \exists x\, \neg Q(x),\ \forall x\, (R(x) \rightarrow \neg P(x))\ \vdash\ \exists x\, \neg R(x)$

[6] $\boxed{4}$ (a) Use resolution to determine satisfiability of the CNF

$$\phi = (\neg p \vee q \vee r) \wedge (p \vee \neg q \vee r) \wedge (p \vee q)$$

[7] (b) Determine satifiability of the Horn formula

$$\psi = (p \wedge q \to r) \wedge (\top \to p) \wedge (p \to q) \wedge (r \to \bot)$$

[7] (c) Use DPLL to determine satisfiability of the CNF

$$\chi = (\neg 1 \vee 2 \vee 3) \wedge (\neg 1 \vee \neg 2) \wedge (\neg 1 \vee \neg 3) \wedge (1 \vee 4)$$

starting with the decision 1.

[20] $\boxed{5}$ Determine whether the following statements are true or false. Every correct answer is worth 2 points. For every wrong answer 1 point is subtracted, provided the total number of points is non-negative.

statement
$\neg\forall x\, P(x) \dashv\vdash \neg\exists y\, \neg P(y)$
Every unary affine boolean function is monotone.
The boolean function $f(x, y) = x \oplus y \oplus \overline{x}$ is affine.
Every Skolem normal form is a prenex normal form.
The set of propositional connectives $\{\top, \neg, \oplus\}$ is adequate.
There exists a sorting network for 4 wires with 4 comparators.
There exists an LTL formula without equivalent CTL formula.
The CTL formulas $\mathsf{A}[\phi\, \mathsf{U}\, \psi]$ and $\neg(\mathsf{E}[\neg\psi\, \mathsf{U}\, (\neg\phi \wedge \neg\psi)] \vee \mathsf{EG}\, \psi)$ are equivalent.
The disjunction of all decision literals when a conflict occurs is a backjump clause in basic DPLL.
The set $\{\mathsf{U}, \mathsf{R}\}$ is an adequate set of connectives for the LTL fragment consisting of negation-normal forms without X.