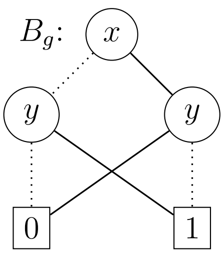
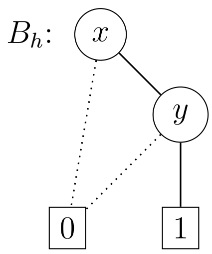
universität innsbruck

Logik	SS 2020	LVA 703027
EXAM 3		February 25, 2021

This exam consists of five exercises. The available points for each item are written in the margin. You need at least 50 points to pass.

1 Consider the boolean function f defined by $f(x, y, z) = xz \oplus yz \oplus \overline{x}\,\overline{z} \oplus y\overline{z}$ and the following two reduced OBDDs:





[6] (a) Construct a reduced OBDD for f with variable ordering $[x, y, z]$.

[7] (b) Compute $\mathsf{apply}(\cdot, B_g, B_h)$.

[7] (c) i. Show that the set containing only f is not adequate.

ii. Determine adequacy of the sets $\{f, \oplus\}$ and $\{f, \cdot\}$.

[7] **2** (a) Use Tseitin's transformation to compute an equisatisfiable CNF of the following propositional formula:

$$\varphi = ((p \vee r) \rightarrow q) \vee \neg(q \wedge \neg p)$$

[7] (b) Use DPLL to determine satisfiability of the CNF

$$\psi = (2 \vee 1 \vee \neg 3) \wedge (1 \vee 3) \wedge (\neg 2 \vee \neg 4) \wedge (\neg 1 \vee 3) \wedge (\neg 1 \vee \neg 2) \wedge (\neg 1 \vee \neg 3 \vee 4)$$

[6] (c) Determine satisfiability of the Horn formula

$$\chi = (q \rightarrow r) \wedge (\top \rightarrow t) \wedge (q \wedge s \rightarrow u) \wedge (\top \rightarrow q) \wedge (u \wedge t \rightarrow \bot) \wedge (r \wedge t \rightarrow s)$$

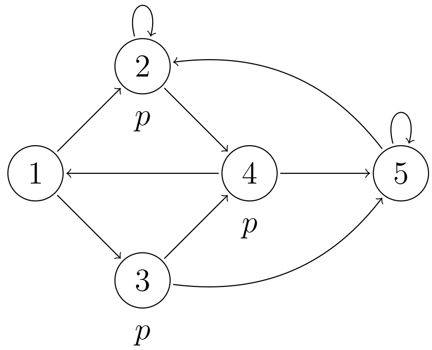
3 For each of the following sequents, either give a natural deduction proof or find a model which does not satisfy it.

[6] (a) $\neg(p \vee q) \vdash r \rightarrow p \rightarrow \top$

[7] (b) $\forall x\, \neg(P(x) \vee Q(x)) \vdash a = b \rightarrow P(a) \wedge Q(b)$

[7] (c) $\forall x\, \neg(P(x) \vee Q(x)) \vdash a = b \rightarrow f(a) = f(b)$

4 Consider the model $\mathcal{M}$:



[7] (a) Use the CTL model checking algorithm to determine in which states of $\mathcal{M}$ the CTL formula $\phi = \mathsf{AX}(\mathsf{A}[p\,\mathsf{U}\,\mathsf{E}[p\,\mathsf{U}\,\neg p]])$ holds.

[6] (b) Give an LTL formula ψ that distinguishes states 3 and 4. (So the formula ψ must hold in exactly one of the two states.)

[7] (c) Are the CTL* formulas $\mathsf{E}[\mathsf{A}[\mathsf{X}\,q]]$ and $\mathsf{A}[\mathsf{E}[\mathsf{A}[\mathsf{X}\,q]]]$ equivalent?

[20] 5 Determine whether the following statements are true or false. Every correct answer is worth 2 points. For every wrong answer 1 point is subtracted, provided the total number of points is non-negative.

statement
Every Horn clause is satisfiable.
$\exists x\,(P(x) \to Q(x)) \vDash \exists x\,P(x) \to \exists x\,Q(x)$
The CTL formula $\mathsf{AG}(p \to \mathsf{AF}\,q)$ is expressible in LTL.
The propositional formula $p \vee (\neg p \wedge q \to p \vee q)$ is valid.
The boolean function $g(x,y,z) = \overline{(x \oplus z)} \oplus \overline{y}$ is adequate.
The term $f(x,z)$ is free for x in $\exists x\,\forall z\,(Q(x) \vee P(z)) \wedge \forall y\,(Q(y) \vee P(x))$.
The set $\{\mathsf{EF}, \mathsf{EU}, \mathsf{EX}\}$ is an adequate set of temporal connectives for CTL.
The boolean function $f(x_1,x_2,x_3,x_4) = x_1 \oplus (x_2 + x_1x_4) \oplus (x_3 + \overline{x_4})$ is monotone.
For every $n > 1$ there exists a sorting network for n wires with at most $2n$ comparators.
The clause $\{Q(y), \neg Q(a)\}$ is a resolvent of the clauses $\{\neg P(x), Q(y)\}$ and $\{P(a), \neg Q(x)\}$.