universität innsbruck

Logik	SS 2021	LVA 703027

EXAM 1	June 21, 2021

This exam consists of four exercises. The available points for each item are written in the margin. You need at least 50 points to pass. ***Explain your answers!!***

1 Consider the boolean function $f(x,y,z) = (((x \oplus y) \to \overline{z}) \to \overline{y}) \to \overline{x}$. Here $u \to v$ stands for $\overline{u} + v$.

[8] (a) Compute the algebraic normal form of f.

[8] (b) Is f monotone? Is f self-dual?

[9] (c) Determine all minimal adequate subsets of $\{\oplus, f, +, \overline{f}\}$.

[8] **2** (a) Use resolution to determine satisfiability of the formula

$$\varphi \;=\; (p \vee q) \wedge (p \to q \vee r) \wedge (q \to s) \wedge (q \vee \neg r) \wedge \neg s$$

[8] (b) Determine whether the terms $f(g(x, f(y,a)), f(a,y))$ and $f(g(z,x),x)$ are unifiable and compute a most general unifier if possible. Here x, y, z are variables and a is a constant.

[9] (c) Transform the following formula into an equisatisfiable Skolem normal form:

$$\psi \;=\; (\forall x\, \forall y\, (P(x,y) \to Q(x))) \to (\forall x\, Q(x))$$

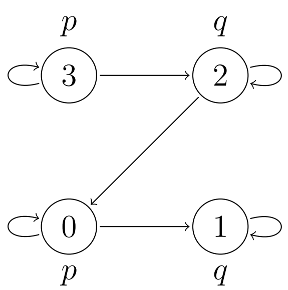
3 For each of the following sequents, either give a natural deduction proof or find a model which does not satisfy it.

[8] (a) $\vdash\; (q \to p) \vee (\neg p \vee r)$

[9] (b) $\exists x\, (P(x) \wedge Q(x)),\ \forall x\, (P(x) \to R(x)) \;\vdash\; \neg\forall x\, \neg(R(x) \wedge Q(x))$

[8] (c) $\forall x\, \forall y\, (P(x,y) \to P(f(x), f(y))),\ \forall x\, \neg P(x, f(x)),\ \forall x\, P(x,x) \;\vdash\; \exists x\, P(f(x), x)$

4 Consider the model $\mathcal{M}$:



[9] (a) Use the CTL model checking algorithm to determine in which states of $\mathcal{M}$ the CTL formula $\varphi = \mathsf{E}[\mathsf{AX}\, p\ \mathsf{U}\ \mathsf{AG}\, q]$ holds.

[9] (b) Determine in which states of $\mathcal{M}$ the LTL formulas $\psi_1 = \mathsf{GF}q$, $\psi_2 = \mathsf{FG}q$, and $\psi_3 = \mathsf{XX}q$ hold and in which not.

[8] (c) For each $0 \leqslant i \leqslant 3$ find a CTL formula χ_i which is only satisfied in state i of $\mathcal{M}$.