universität innsbruck

Logik	SS 2021	LVA 703027
EXAM 2		September 27, 2021

This exam consists of four exercises. The available points for each item are written in the margin. You need at least 50 points to pass. ***Explain your answers!!***

1 Consider the boolean function f defined by $f(x, y, z) = (\overline{x} \oplus yz \oplus xyz) + x\overline{y}$.

[6] (a) Construct a reduced OBDD for f with variable ordering $[x, y, z]$.

[6] (b) Compute the algebraic normal form of f.

[6] (c) Give an expression only using f and the variable x that is equivalent to the boolean function $g(x) = 1$.

[7] (d) Is $\{f\}$ adequate? Prove your answer!

2 Consider the propositional formula

$$\varphi \;=\; (q \vee r) \wedge (\neg r \vee \neg p) \wedge (s \to p \vee q) \wedge (q \to \neg p) \wedge (q \to s) \wedge (\neg s \vee p) \wedge (q \vee s)$$

[8] (a) Use resolution to determine satisfiability of φ.

[8] (b) Use DPLL to determine satisfiability of φ.

[9] (c) Use resolution to determine satisfiability of the clausal form

$$\{\{\neg Q(a)\}, \{P(f(z), a)\}, \{\neg P(x, y), P(y, x)\}, \{\neg P(u, f(u)), Q(u), Q(v)\}\}$$

where x, y, z, u, v are variables and a is a constant.

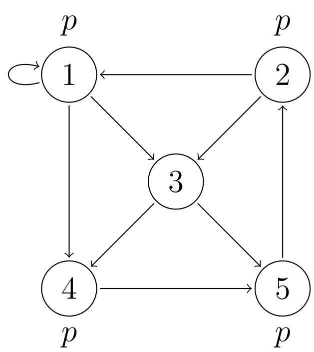
3 For each of the following sequents, either give a natural deduction proof or find a model which does not satisfy it.

[8] (a) $p \to q,\ r \to q \ \vdash\ p \vee r \vee \neg q$

[9] (b) $\exists x\ (P(x) \to Q(x)),\ \exists x\ P(x) \ \vdash\ \exists x\ Q(x)$

[8] (c) $\exists x\ ((P(x) \to Q(x)) \wedge P(x)) \ \vdash\ \exists x\ Q(x)$

4 Consider the model $\mathcal{M}$:



[9] (a) Use the CTL model checking algorithm to determine in which states of $\mathcal{M}$ the CTL formula $\varphi = \mathsf{AX}\,\mathsf{A}[\mathsf{EG}\,p\ \mathsf{U}\ \mathsf{AX}\,p]$ holds.

[8] (b) Construct an LTL formula ψ that distinguishes states 4 and 5.

[8] (c) Construct a CTL formula χ that distinguishes states 1 and 2.