universität innsbruck

Logik	SS 2021	LVA 703027
EXAM 3		February 24, 2022

This exam consists of four exercises. The available points for each item are written in the margin. You need at least 50 points to pass. ***Explain your answers!!***

1 Consider the following two reduced OBDDs:

[8] (a) Compute $\mathsf{apply}(\cdot, B_f, B_g)$ using the variable ordering $[x, y, z]$.

[9] (b) Compute the algebraic normal form for f and determine if f is affine.

[8] (c) Determine if g is (i) monotone, (ii) self-dual, and (iii) affine.

[9] 2 (a) Use resolution to determine whether the formula

$$\varphi = ((p \to q) \to p) \to p$$

is valid or not.

[8] (b) Use resolution to determine whether the formula

$$\psi = (P(a) \lor Q(f(a))) \land \forall x\,((R(x) \to Q(x)) \land (P(x) \to R(f(x))))$$

is satisfiable or not. Here a is a constant.

[8] (c) Transform the following formula χ into an equisatisfiable Skolem normal form:

$$\exists x\,(\forall y\,(P(x,y) \to \exists z\,Q(x, f(z))) \to (\exists y\,P(y,z)))$$

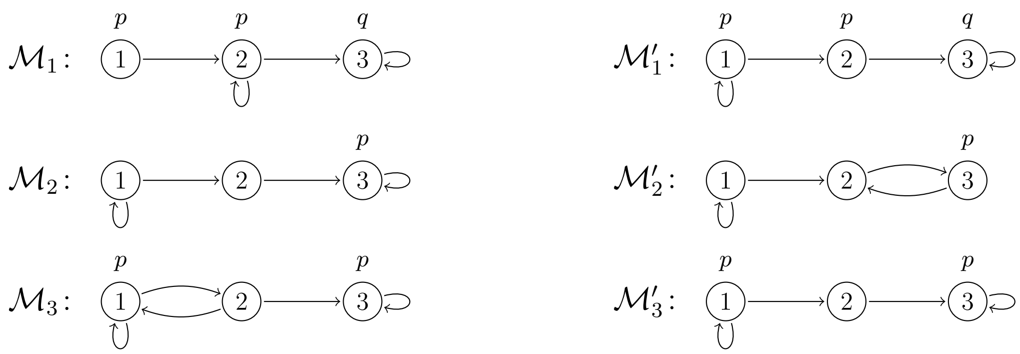
3 For each of the following sequents, either give a natural deduction proof or find a model which does not satisfy it.

[9] (a) $p \land q \to r,\ \neg r,\ p\ \vdash\ q$

[8] (b) $\vdash\ \forall x\,\exists y\,P(x,y) \to \exists x\,P(x,x)$

[8] (c) $\vdash\ \neg(\exists x\,\forall y\,(\neg P(x) \land P(y)))$

4 Consider the following six models:



[8] (a) Use the CTL model checking algorithm to determine in which states of $\mathcal{M}_1$ the CTL formula $\varphi = \mathsf{E}[\mathsf{AX}\, p \;\mathsf{U}\; \mathsf{AG}\, q]$ holds.

[9] (b) For all $1 \leqslant i \leqslant 3$ find a CTL formula ψ_i such that $\mathcal{M}_i, 1 \vDash \psi_i$ and $\mathcal{M}'_i, 1 \nvDash \psi_i$.

[8] (c) Find an LTL formula χ such that $\mathcal{M}_2, 1 \vDash \chi$ and $\mathcal{M}'_2, 1 \nvDash \chi$.