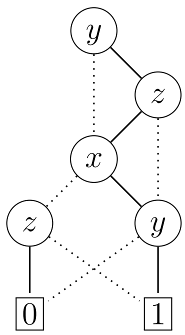
universität innsbruck

Logik	SS 2022	LVA 703027
EXAM 1		June 27, 2022

This exam consists of five exercises. The available points for each item are written in the margin. You need at least 50 points to pass. ***Explain your answers to the first four exercises!***

1 Consider the boolean function $f(x, y, z) = \overline{x}y + x\overline{y}\,\overline{z}$ and the BDD B_g



[6] (a) Is f monotone? Is f self-dual?

[8] (b) Compute the algebraic normal forms of f and g.

[6] (c) Can $\overline{x}$ be expressed using f, g and the variable x?

[6] 2 (a) Determine whether the terms $f(g(x, y), h(z, z))$ and $f(g(g(w, z), a), w)$ are unifiable and compute a most general unifier if possible. Here, w, x, y, z are variables and a is a constant.

[7] (b) Transform the following formula into an equisatisfiable Skolem normal form:

$$\psi = \forall x\, \exists y\, \big(P(x, y) \wedge (\forall z\, (P(x, z) \wedge P(z, y))) \rightarrow Q(x, y)\big)$$

[7] (c) Use resolution to determine satisfiability of the clausal form

$$\{\{P(x),\, P(f(x))\},\, \{\neg P(x),\, \neg P(f(f(x)))\}\}$$

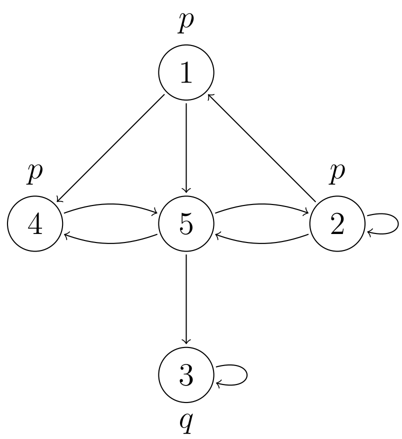
3 For each of the following sequents, either give a natural deduction proof or find a model which does not satisfy it.

[6] (a) $\neg x \wedge \neg y \ \vdash\ \neg(x \vee y)$

[7] (b) $\forall x\, (P(x) \vee Q(x)),\, \exists x\, \neg P(x) \ \vdash\ \forall x\, (R(x) \rightarrow \neg Q(x)) \rightarrow \exists x\, \neg R(x)$

[7] (c) $\forall x\, (P(x) \vee Q(x)),\, \exists x\, \neg P(x) \ \vdash\ \forall x\, (R(x) \rightarrow \neg P(x)) \rightarrow \exists x\, \neg R(x)$

4 Consider the model $\mathcal{M}$:



[6] (a) Use the CTL model checking algorithm to determine in which states of $\mathcal{M}$ the CTL formula $\varphi = \mathsf{EX}\,\mathsf{A}[\mathsf{EF}\,p\ \mathsf{U}\ \mathsf{AG}\,q]$ holds.

[7] (b) Construct an LTL formula ψ that distinguishes states 1 and 2.

[7] (c) Transform φ into an equivalent CTL formula that uses only temporal connectives from $\{\mathsf{AX}, \mathsf{EG}, \mathsf{EU}\}$.

[20] 5 Determine whether the following statements are true or false. Every correct answer is worth 2 points. For every wrong answer 1 point is subtracted, provided the total number of points is non-negative.

statement
Propositional resolution always terminates.
The LTL formula α is equivalent to the CTL* formula $\mathsf{A}[\alpha]$.
Every predicate logic formula has an equivalent prenex normal form.
A predicate logic formula ϕ is valid if and only if $\neg\phi$ is *not* satisfiable.
For every boolean function f and variable x, $f = \overline{x} \cdot f[0/x] \oplus x \cdot f[1/x]$.
The set $\{\leftrightarrow, \neg\}$ is an adequate set of connectives for propositional logic.
There exists an algorithm for deciding validity of predicate logic formulas.
The set $\{\mathsf{EX}, \mathsf{EG}, \mathsf{AU}\}$ is an adequate set of temporal connectives for CTL.
For every propositional formula an equivalent CNF can be computed in linear time.
The substitution $\{x \mapsto f(a,a),\ y \mapsto a\}$ is a most general unifier of the terms $f(a,y)$ and x.