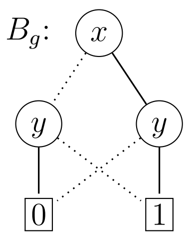
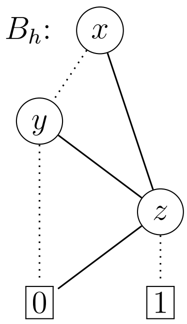
universität innsbruck

Logik	SS 2022	LVA 703027
EXAM 3		February 23, 2023

This exam consists of five exercises. The available points for each item are written in the margin. You need at least 50 points to pass. ***Explain your answers to the first four exercises!***

1 Consider the boolean function f defined by $f(x, y, z) = x(y \oplus z) \oplus (x + \overline{z})$ and the following two reduced OBDDs:





[6] (a) Construct a reduced OBDD for f with variable ordering $[x, y, z]$.

[7] (b) Compute $\mathsf{apply}(\oplus, B_g, B_h)$.

[7] (c) Determine which of the five properties from Post's adequacy theorem hold for f. Is the set $\{0, f\}$ adequate?

[7] **2** (a) Use DPLL to determine satisfiability of

$$\varphi = (s \wedge r \to q) \wedge (\neg p \to \neg s) \wedge (\neg p \vee r) \wedge (\neg p \vee \neg q) \wedge (s \vee p) \wedge (\neg q \to s \vee \neg r)$$

[7] (b) Determine satisfiability of the Horn formula

$$\psi = (r \wedge s \to q) \wedge (q \wedge u \to t) \wedge (\top \to p) \wedge (p \wedge r \to s) \wedge (t \wedge r \to \bot) \wedge (p \to r)$$

[6] (c) Transform the following formula χ into an equisatisfiable Skolem normal form:

$$\chi = \exists x\, \big(\exists y\, P(x, y) \to \exists z\, H(f(z))\big) \to \forall z\, Q(z)$$

3 For each of the following sequents, either give a natural deduction proof or find a model which does not satisfy it.

[6] (a) $\vdash\ \forall x\, (x = a \to x = b) \to a = b$

[7] (b) $\forall x\, (P(x) \to Q(x))\ \vdash\ \exists x\, P(x) \to \forall x\, Q(x)$

[7] (c) $\forall x\, (P(x) \vee Q(x))\ \vdash\ \forall x\, P(x) \vee \exists x\, Q(x)$

4 This exercise is about the construction of the labeled Büchi automaton $A_{\neg\varphi}$ in the context of the LTL model checking algorithm. Let φ be an arbitrary LTL formula.

[4] (a) The closure $\mathcal{C}(\varphi)$ of φ consists of all subformulas φ and there negations, where $\neg\neg\psi$ and ψ are identified. Complete the following definition:

The states of the automaton $A_{\neg\varphi}$ are the maximal (with respect to $\subseteq$) subsets S of $\mathcal{C}(\varphi)$ such that

1. for all non-negated $\psi \in \mathcal{C}(\varphi)$ either $\psi \in S$ or $\boxed{}^{\mathbf{1}}$
2. $\psi_1 \vee \psi_2 \in \mathcal{C}(\varphi) \implies (\psi_1 \vee \psi_2 \in S \iff \psi_1 \in S \text{ or } \psi_2 \in S)$
3. $\psi_1 \wedge \psi_2 \in \mathcal{C}(\varphi) \implies (\psi_1 \wedge \psi_2 \in S \iff \psi_1 \in S \text{ and } \psi_2 \in S)$
4. $\psi_1 \to \psi_2 \in \mathcal{C}(\varphi) \implies (\psi_1 \to \psi_2 \in S \iff \boxed{}^{\mathbf{2}})$
5. $\psi_1 \,\mathsf{U}\, \psi_2 \in S \implies \boxed{}^{\mathbf{3}}$ or $\psi_2 \in S$
6. $\boxed{}^{\mathbf{4}} \implies \neg\psi_2 \in S$

[4] (b) Complete the following definition of the transition relation δ of $A_{\neg\varphi}$:

$(S, T) \in \delta$ if and only if the following conditions are satisfied:

1. $\mathsf{X}\,\psi \in S \implies \psi \in T$
2. $\neg\mathsf{X}\,\psi \in S \implies \boxed{}^{\mathbf{1}}$
3. $\psi_1 \,\mathsf{U}\, \psi_2 \in S$ and $\boxed{}^{\mathbf{2}} \implies \psi_1 \,\mathsf{U}\, \psi_2 \in T$
4. $\neg(\psi_1 \,\mathsf{U}\, \psi_2) \in S$ and $\boxed{}^{\mathbf{3}} \implies \boxed{}^{\mathbf{4}}$

[4] (c) What are the initial states of $A_{\neg\varphi}$? A trace t is an infinite sequence of valuations of propositional atoms. When is t accepted by $A_{\neg\varphi}$?

[8] (d) Construct $A_{\neg\chi}$ for the LTL formula $\chi = p \,\mathsf{U}\, (\neg\mathsf{X}\, p)$ and determine which of the following traces are accepted by $A_{\neg\chi}$:

1. $\{p\}^\omega$
2. $\{p\}\varnothing\{p\}^\omega$
3. $\varnothing\{p\}\varnothing\{p\}^\omega$

[20] $\boxed{5}$ Determine whether the following statements are true or false. Every correct answer is worth 2 points. For every wrong answer 1 point is subtracted, provided the total number of points is non-negative.

statement
Propositional resolution is sound and complete.
The CTL formula $\mathsf{AG}\,\mathsf{EF}\,q$ can be expressed in LTL.
The formulas $(p \vee q) \wedge \neg p$ and $q \wedge \neg q$ are equisatisfiable.
The term $f(y)$ is free for x in $\forall x\, \exists y\, (P(x) \wedge P(y) \wedge P(z))$.
Satisfaction of LTL formulas in finite models is undecidable.
Every propositional formula ϕ has a unique conjunctive normal form.
The LTL formulas $\mathsf{G}(\phi \wedge \neg\psi)$ and $\mathsf{G}\,\phi \wedge \neg\,\mathsf{F}\,\psi$ are semantically equivalent.
The set $\{\mathsf{EX}, \mathsf{EG}, \mathsf{EU}\}$ is an adequate set of temporal connectives for CTL.
Every boolean function $f\colon \{0,1\}^n \to \{0,1\}$ has a unique algebraic normal form.
If two terms s and t are not unifiable, then the unification algorithm on $s \approx t$ does not terminate.