universität innsbruck

Logik	SS 2023	LVA 703027
EXAM 1		June 26, 2023

This exam consists of five exercises. The available points for each item are written in the margin. You need at least 50 points to pass. ***Explain your answers to the first four exercises!***

1 Consider the three boolean functions f_1, f_2 and f_3 defined as follows:

$$f_i(x_1, x_2, x_3) = \begin{cases} x_1 & \text{if } s = 0 \\ x_i & \text{if } s = 1 \\ x_s & \text{if } s > 1 \end{cases}$$

for $i \in \{1, 2, 3\}$. Here $s = x_1 + x_2 + x_3$ is the sum of the inputs, which evaluates to a natural number between 0 and 3.

[7] (a) Compute the algebraic normal forms of f_1, f_2 and f_3.

[7] (b) Is f_1 monotone? Is f_2 self-dual? Is f_3 affine? Is $\{f_1, f_2, f_3\}$ adequate?

[6] (c) How many different affine boolean functions of arity 3 exist?

[7] **2** (a) Use resolution to determine satisfiability of the CNF

$$\varphi \;=\; (p \vee q \vee r) \wedge (p \vee \neg q \vee r) \wedge (q \vee \neg r) \wedge (\neg p \vee r)$$

[7] (b) Transform the following formula ψ into an equisatisfiable Skolem normal form:

$$\psi \;=\; \neg\big(\forall x\, \exists y\, P(x, f(y, x)) \rightarrow \forall z\, Q(z)\big) \vee \forall x\, R(x, x)$$

[6] (c) Use the unification algorithm to determine whether the terms $f(f(a, g(x)), f(g(y), z))$ and $f(f(z, w), f(g(x), w))$ are unifiable and compute a most general unifier if possible. Here x, y, z, w are variables and a is a constant.

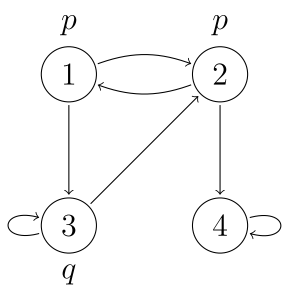
3 For each of the following sequents, either give a natural deduction proof or find a model which does not satisfy it.

[7] (a) $\forall x\, Q(x, g(x)) \;\vdash\; \exists y\, \forall x\, Q(x, y)$

[6] (b) $p \rightarrow \neg r,\ q \rightarrow \neg r \;\vdash\; r \rightarrow \neg(q \vee p)$

[7] (c) $\forall x\, (Q(a, x) \rightarrow a = x) \;\vdash\; \forall x\, (Q(a, b) \wedge P(a, x) \rightarrow P(b, x))$

4 Consider the model $\mathcal{M}$:



[6] (a) Use the CTL model checking algorithm to determine in which states of $\mathcal{M}$ the CTL formula $\varphi = \mathsf{AX}\,\mathsf{E}[p\,\mathsf{U}\,\mathsf{AG}\,\neg q]$ holds.

[7] (b) Determine in which states of $\mathcal{M}$ the LTL formulas $\psi_1 = \mathsf{F}\,\mathsf{X}(p \vee q)$, $\psi_2 = p \to \mathsf{G}\,q$, and $\psi_3 = q\,\mathsf{U}\,p$ hold.

[7] (c) Give a CTL formula without the connectives AX and AG that is equivalent to $\mathsf{AX}\,\mathsf{AG}\,p$.

[20] 5 Determine whether the following statements are true or false. Every correct answer is worth 2 points. For every wrong answer 1 point is subtracted, provided the total number of points is non-negative.

statement
The sequent $\bot \vdash p \to q$ is valid.
Every Skolem normal form is a prenex normal form.
The term $f(x,y)$ is free for x in $\forall x\, P(f(x,y)) \vee \forall y\, P(y)$.
The LTL formulas $\mathsf{G}(\varphi \vee \psi)$ and $\mathsf{G}\,\varphi \vee \mathsf{G}\,\psi$ are semantically equivalent.
The set $\{\mathsf{AX}, \mathsf{AF}, \mathsf{AG}\}$ is an adequate set of temporal connectives for CTL.
An adequate set of boolean functions cannot contain any affine boolean functions.
The reduced OBDD representation of a boolean function for a given variable order is unique.
Every propositional formula can be transformed into an equisatisfiable DNF using Tseitin's transformation.
The substitution $\{z \mapsto g(x,y),\, y \mapsto g(x,y),\, x \mapsto h(g(x,y))\}$ is a unifier of the terms $f(g(x,y),y,x)$ and $f(z,z,h(z))$.
If there exists a model which satisfies a given predicate logic formula φ then there must also exist a natural deduction proof for φ.