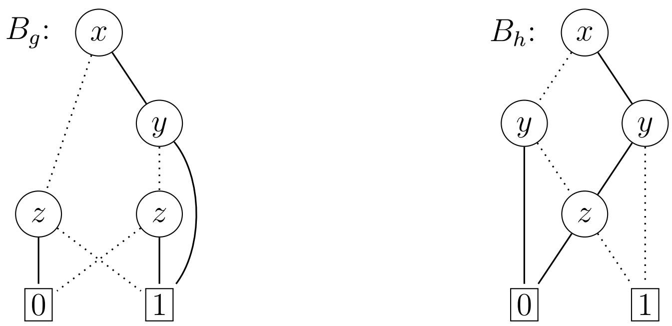
Logik | SS 2023 | LVA 703027

EXAM 2 | September 27, 2023

This exam consists of five exercises. The available points for each item are written in the margin. You need at least 50 points to pass. ***Explain your answers to the first four exercises!***

1 Consider the boolean function f defined by $f(x, y, z) = \overline{x}\,\overline{y} + \overline{x}z + \overline{y}z$ and the following two reduced OBDDs:



[6] (a) Transform B_g into an equivalent reduced OBDD with variable ordering $[y, z, x]$.

[7] (b) Compute $\mathsf{apply}(\cdot, B_g, B_h)$.

[7] (c) Show that the set $\{f, \overline{}\}$ is not adequate.

[6] **2** (a) Using the unification algorithm, determine if the terms

$$g(x, f(y), h(z, f(z))) \quad \text{and} \quad g(h(a, y), f(f(a)), x)$$

are unifiable. If they are unifiable, find the most general unifier. Here a is a constant and x, y and z are variables.

[7] (b) Transform the following sentence into an equisatisfiable Skolem normal form:

$$\varphi = (\forall x\, \exists y\, P(x, f(y, x)) \wedge \neg \forall z\, Q(z)) \rightarrow \forall x\, \neg \forall y\, R(x, y)$$

[7] (c) Use resolution to determine satisfiability of the clausal form

$$\{\{Q(x), Q(f(a))\}, \{P(f(a), a)\}, \{\neg P(x, y), P(f(y), x)\}, \{\neg P(f(x), y), \neg Q(y)\}\}$$

where a is a constant and x, y and z are variables.

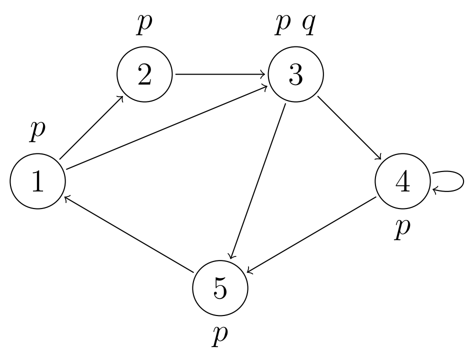
3 For each of the following sequents, either give a natural deduction proof or find a model which does not satisfy it.

[7] (a) $\vdash\ ((p \vee \neg q) \wedge (\neg r \rightarrow q)) \rightarrow (r \vee p)$

[6] (b) $\forall x\, (P(x) \wedge (\forall y\, \neg Q(x, y) \rightarrow \neg P(x)))\ \vdash\ \forall y\, \exists x\, Q(x, y)$

[7] (c) $\exists x\, \forall y\, \neg P(x, y), \forall z\, (a = a \rightarrow \neg Q(z))\ \vdash\ \neg \forall x\, \exists y\, (P(x, y) \vee Q(x))$

4 Consider the model $\mathcal{M}$:



[6] (a) Use the CTL model checking algorithm to determine in which states of $\mathcal{M}$ the CTL formula $\varphi = \mathsf{EX}\,\mathsf{A}[\neg q \,\mathsf{U}\, \mathsf{EX}\, q]$ holds.

[6] (b) Determine in which states of $\mathcal{M}$ the LTL formula $\psi = (\mathsf{X}\,\neg p)\;\mathsf{U}\;(\mathsf{X}\,q)$ holds.

[8] (c) For each $1 \leqslant i \leqslant 5$ find a CTL formula χ_i which holds only in state i of $\mathcal{M}$.

[20] 5 Determine whether the following statements are true or false. Every correct answer is worth 2 points. For every wrong answer 1 point is subtracted, provided the total number of points is non-negative.

statement
$\neg(\varphi \leftrightarrow \psi) \equiv (\varphi \leftrightarrow \neg\psi)$
The set $\{+, \oplus\}$ is adequate.
Validity of CNFs is efficiently decidable.
The empty clause is a resolvent of $\{\neg p, q\}$ and $\{p, \neg q\}$.
The CTL formulas $\mathsf{AG}\,p$ and $\neg\,\mathsf{EF}\,\neg\,\mathsf{AG}\,p$ are equivalent.
Tseitin's transformation produces an equivalent formula.
The DPLL algorithm is used to get all satisfying assignments of a propositional formula.
For every formula in predicate logic, there is an equivalent formula in prenex normal form.
For any given variable ordering, $(A \vee B) \wedge C$ and $(A \wedge C) \vee (B \wedge C)$ result in the same reduced OBDD.
The following statement is known as soundness in propositional logic: $\varphi_1, \varphi_2, \ldots, \varphi_n \vdash \psi$ is valid $\implies$ $\varphi_1, \varphi_2, \ldots, \varphi_n \vDash \psi$