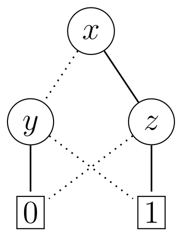
Logik	SS 2023	LVA 703027
EXAM 3		February 23, 2024

This exam consists of five exercises. The available points for each item are written in the margin. You need at least 50 points to pass. ***Explain your answers to the first four exercises!***

1 Consider the boolean function $f(x, y, z) = (x \oplus y) + z$ and the BDD B_g



[7] (a) Starting from B_g, compute a reduced OBDD that is equivalent to $\exists y.g$.

[6] (b) Compute the algebraic normal forms of f and g.

[7] (c) Determine which of the five properties from Post's adequacy theorem hold for f. Which of the three sets $\{f, \overline{}\}$, $\{\overline{f}\}$ and $\{f, 1\}$ are adequate?

[6] 2 (a) Use Tseitin's transformation to compute an equisatisfiable CNF of the propositional formula

$$\varphi \;=\; q \vee (p \rightarrow s) \rightarrow \neg p \vee s$$

[7] (b) Use DPLL with first decision $\overset{d}{p}$ to determine satisfiability of

$$\psi \;=\; (p \vee s) \wedge (p \rightarrow \neg q) \wedge (\neg q \rightarrow r) \wedge (\neg p \vee \neg r \vee s) \wedge (q \vee \neg r \vee \neg s)$$

[7] (c) Use resolution to determine satisfiability of the following clausal form, where a is a constant and x is a variable:

$$\{\{P(f(f(a)))\}, \{\neg P(x), Q(x)\}, \{P(x), R(f(x))\}, \{\neg Q(f(a))\}, \{\neg Q(x), \neg R(x)\}\}$$

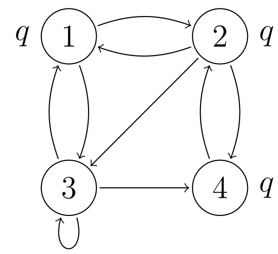
3 For each of the following sequents, either give a natural deduction proof or find a model which does not satisfy it.

[6] (a) $\vdash\; p \vee (p \rightarrow q)$

[7] (b) $\forall x\, (\exists y\, R(x, y) \rightarrow \neg R(x, x)), \exists x\, \forall y\, R(y, x) \;\vdash\; \forall x\, \neg R(x, x)$

[7] (c) $\forall x\, \exists y\, (R(x, y) \wedge P(y)), \exists x\, \neg R(x, x) \;\vdash\; \neg \forall x\, \forall y\, (P(x) \wedge P(y) \rightarrow x = y)$

4 Consider the following model $\mathcal{M}$:



[6] (a) Determine in which states of $\mathcal{M}$ the CTL formula

$$\varphi \;=\; \mathsf{E}[\mathsf{EG}\, q \wedge \mathsf{EX}\,\neg q \,\mathsf{U}\, \mathsf{AX}\, q]$$

is satisfied by applying the CTL model checking algorithm.

[7] (b) Find a CTL formula ψ such that $\mathcal{M}, 1 \vDash \psi$ and $\mathcal{M}, 2 \nvDash \psi$.

[7] (c) Find an LTL formula χ such that neither $\mathcal{M}, 2 \vDash \chi$ nor $\mathcal{M}, 2 \vDash \neg\chi$.

[20] 5 Determine whether the following statements are true or false. Every correct answer is worth 2 points. For every wrong answer 1 point is subtracted, provided the total number of points is non-negative.

statement
$[\![\mathsf{EG}\,\varphi]\!] = [\![\varphi]\!] \cap \mathsf{pre}_\exists([\![\mathsf{EG}\,\varphi]\!])$
The proof rules ¬¬i and PBC are inter-derivable.
The dual of a monotone boolean function is monotone.
The CTL* formulas $\mathsf{E}[\mathsf{X}\,\mathsf{F}\,\mathsf{G}\,\neg p]$ and $\neg\mathsf{A}[\mathsf{X}\,\mathsf{G}\,\mathsf{E}[\mathsf{F}\,p]]$ are semantically equivalent.
Every propositional CNF formula is semantically equivalent to a Horn formula.
The clausal form $\{\{p, \neg q\}, \{q, p, r\}, \{\neg r, q\}, \{\neg q\}\}$ has four different resolvents.
The term $f(x, z)$ is free for y in the formula $\forall x\, (\exists y\, (x = y \wedge P(x)) \vee \exists z\, Q(x, z))$.
The formulas $(p \vee q) \wedge (q \to \neg(r \vee p))$ and $((p \to q) \to r) \to p$ are equisatisfiable.
The CTL formulas $\mathsf{A}[\varphi \,\mathsf{U}\, \psi]$ and $\mathsf{AF}\,\psi \wedge \neg\mathsf{E}[\neg\psi \,\mathsf{U}\, (\neg\varphi \wedge \neg\psi)]$ are semantically equivalent.
If a set X of boolean functions is not adequate then $\overline{x}$ or $x \oplus y$ is not expressible using functions from X.