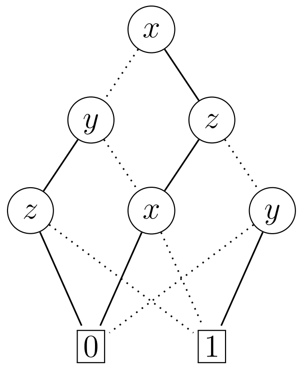
universität innsbruck

Logik	SS 2024	LVA 703026
EXAM 1		June 24, 2024

This exam consists of five exercises. The available points for each item are written in the margin. You need at least 50 points to pass. ***Explain your answers to the first four exercises!***

1 Consider the boolean function $f(x,y,z) = (x \oplus \overline{y}) \cdot (\overline{x} + z)$ and the BDD B_g



[2] (a) Is B_g reduced? Is B_g ordered?

[5] (b) Transform B_g into an equivalent reduced OBDD with variable ordering $[x,z,y]$.

[5] (c) Compute the algebraic normal forms of f and g.

[5] (d) Which of the properties from Post's adequacy theorem hold for f and g?

[3] (e) Which subsets of $\{f, g, \oplus, +\}$ are adequate?

[6] **2** (a) Determine whether the terms $f(h(z), g(x,x), z)$ and $f(h(x), y, h(y))$ are unifiable and compute a most general unifier if possible. Here, x, y and z are variables.

[7] (b) Transform the following formula into an equisatisfiable Skolem normal form:

$$\varphi \;=\; \forall x\, \exists y\, (P(x) \to P(y)) \to \forall y\, \exists x\, Q(x,y)$$

[7] (c) Use resolution to determine satisfiability of the clausal form

$$\{\, \{Q(x),\, \neg P(f(x), f(x))\},\, \{R(f(x), y),\, Q(x)\},\, \{\neg Q(a)\},\, \{R(x, f(y)),\, R(f(u), v)\}, \{P(x,y),\, \neg R(x,y)\}\,\}$$

Here a is a constant.

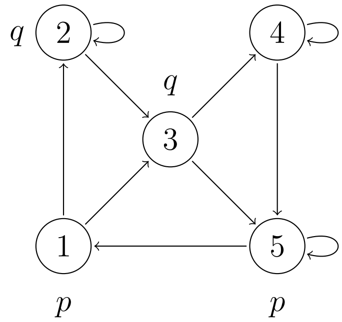
3 For each of the following sequents, either give a natural deduction proof or find a model which does not satisfy it.

[6] (a) $\neg(\neg q \wedge p) \;\vdash\; q \vee \neg p$

[7] (b) $\forall x\, R(x) \vee \forall x\, \exists y\, S(x,y) \;\vdash\; \forall x\, \exists y\, (S(x,y) \vee R(x))$

[7] (c) $\forall x\, R(x) \wedge \forall x\, \exists y\, S(x,y) \;\vdash\; \exists y\, \forall x\, (S(x,y) \wedge R(x))$

4 Consider the following model $\mathcal{M}$:



[6] (a) Use the CTL model checking algorithm to determine in which states of $\mathcal{M}$ the CTL formula $\varphi = \mathsf{EG}((p \vee q) \wedge (q \rightarrow \mathsf{EX}\,\neg q))$ holds.

[7] (b) Specify a path which satisfies LTL formula $q \wedge \mathsf{X}(\neg q \wedge (p\,\mathsf{U}\,q))$.

[7] (c) For each $1 \leqslant i \leqslant 5$ find a CTL formula χ_i without AX and EX which holds only in state i of $\mathcal{M}$.

[20] 5 Determine whether the following statements are true or false. Every correct answer is worth 2 points. For every wrong answer 1 point is subtracted, provided the total number of points is non-negative.

statement
The sequent $p \rightarrow q \vdash \neg p \rightarrow \neg q$ is not valid.
The function $f(x, y) = x \oplus y \oplus \overline{y}$ is monotone.
Resolution is complete but not sound for predicate logic.
Satisfaction of CTL formulas in finite models is decidable.
In DPLL any backjump can always be simulated by a backtrack instead.
It is not possible to verify if some comparator network is a sorting network.
The CTL formula $p \wedge \mathsf{EX}\,\mathsf{EF}\,p$ is semantically equivalent to the LTL formula $\mathsf{F}\,p$.
For every boolean function there exist at least two different reduced BDD representations.
The Skolem normal form of a predicate logic formula cannot contain any existential quantifiers.
The substitution $\{x \mapsto h(z),\ y \mapsto h(z)\}$ is a most general unifier of the terms $f(x, y, g(x))$ and $f(h(z), h(z), g(z))$.