This exam consists of five exercises. The available points for each item are written in the margin. You need at least 50 points to pass. ***Explain your answers to the first four exercises!***

[6] **1** (a) Use Tseitin's transformation to compute an equisatisfiable CNF of the following porpositional formula:

$$\varphi \;=\; \neg(p \wedge q \to r \vee \neg(q \wedge p))$$

[7] (b) Use DPLL to determine satisfiability of the following CNF:

$$\psi \;=\; (\neg q \vee \neg r) \wedge (\neg p \vee r \vee \neg t) \wedge (p \vee r) \wedge (\neg p \vee t) \wedge (q \vee \neg r) \wedge (t \vee \neg r \vee s)$$

[7] (c) Describe a procedure to check by means of DPLL whether or not $\varphi_1, \ldots, \varphi_n \vDash \psi$ for arbitrary propositional formulas $\varphi_1, \ldots, \varphi_n$ and ψ and argue why your proposed procedure is correct. Use your procedure to show that

$$p \vee q \to r,\; t \to p,\; t \;\vDash\; r$$

[7] **2** (a) Use resolution to determine satisfiability of the formula

$$\varphi \;=\; (\neg p \vee \neg r) \wedge (\neg p \to \neg q \vee r) \wedge (\neg r \to q) \wedge (p \to \neg q) \wedge (r \to p)$$

[7] (b) Transform the following formula ψ into an equisatisfiable Skolem normal form:

$$\psi \;=\; \exists x\, (\forall y\, P(g(x), y) \to \forall y\, (\exists z\, R(y, z) \to \exists x\, P(x, g(y))))$$

[6] (c) Determine satisfiablility of the following causal form using resolution, where a is a constant, and x and y are variables.

$$\{\, \{\neg Q(x), \neg P(y, f(x))\}, \{P(f(f(f(a))), f(a))\}, \{P(x, y), \neg P(f(y), f(x))\}, \{Q(f(a))\} \,\}$$

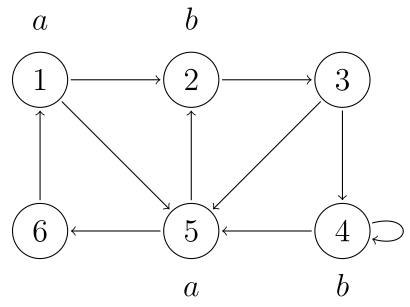
3 For each of the following sequents, either give a natural deduction proof or find a model which does not satisfy it.

[6] (a) $\neg s \vee \neg t,\; \top \to s \wedge t \;\vdash\; \neg s \wedge \neg t$

[7] (b) $\forall x\, \exists y\, (P(x) \to Q(x, y)),\; \neg \exists x\, Q(a, x) \;\vdash\; \neg \forall x\, P(x)$

[7] (c) $\forall x\, \exists y\, (P(x) \to Q(x, y)),\; \neg \forall x\, P(x) \;\vdash\; \neg \exists x\, Q(a, x)$

4 Consider the following model $\mathcal{M}$:



[6] (a) Use the CTL model checking algorithm to determine in which states of $\mathcal{M}$ the CTL formula $\varphi = \mathsf{AX}\,\mathsf{E}[\mathsf{AX}\,a\,\mathsf{U}\,\mathsf{EX}\,b]$ holds.

[6] (b) Find an LTL formula ψ such that neither $\mathcal{M}, 6 \vDash \psi$ nor $\mathcal{M}, 6 \vDash \neg\psi$.

[8] (c) For each $1 \leqslant i \leqslant 6$ find a CTL formula χ_i which holds only in state i of $\mathcal{M}$.

[20] 5 Determine whether the following statements are true or false. Every correct answer is worth 2 points. For every wrong answer 1 point is subtracted, provided the total number of points is non-negative.

statement
The function $p + \overline{q + \overline{p}}$ is affine.
If $\neg\varphi$ is valid then φ is unsatisfiable.
In LTL, if $\pi \vDash p$ then also $\pi \vDash \mathsf{G}\,p\,\mathsf{U}\,\neg p$.
The sequent $\exists x\,\forall y\,Q(x,y) \vdash \forall x\,\exists y\,Q(y,x)$ is valid.
The formula $(\forall x\,P(x)) \to P(x) \vee \exists z\,Q(z)$ is a sentence.
The CTL formula $\mathsf{EF}\,p \vee \mathsf{AF}\,\neg p$ is satisfied in all states of all models.
Any valid propositional formula can be proven using natural deduction.
For a given formula in DNF, it is easier to check validity than satisfiability.
If two terms are unifiable, there is a unique most general unifier which unifies them.
If a comparator network with n inputs sorts 2^n different bit-strings of length n, it is a sorting network.