

Algorithmische Mathematik 7

Logic in Computer Science

This exam consists of four exercises. *Explain your answers.* The available points for each item are written in the margin. You need at least 50 points to pass.

1 Consider the following questions concerning propositional logic.

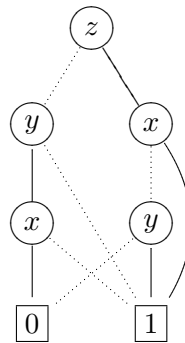
- [8] (a) Give a natural deduction proof of the sequent  $\neg p \rightarrow \neg\neg q \vdash p \vee q$ .
- [8] (b) Transform the formula

$$(\neg p \rightarrow \neg q \wedge \neg r) \wedge (\neg q \rightarrow p \vee r) \wedge (\neg r \rightarrow \neg s \wedge \neg t) \wedge (p \wedge q \rightarrow r) \wedge (r \wedge s \rightarrow \neg q \vee t)$$

into clausal form.

- [8] (c) Is the formula of part (b) satisfiable?

2 Consider the following BDD  $B$ :

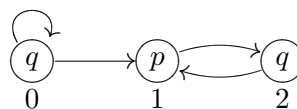


- [7] (a) Is  $B$  reduced?
- [7] (b) Is  $B$  ordered?
- [7] (c) Find an equivalent reduced OBDD with respect to the variable ordering  $[x, y, z]$ .
- [7] (d) Which boolean function does  $B$  represent?

3 For each of the following formulas of predicate logic, either give a natural deduction proof or find a model which does not satisfy it:

- [8] (a)  $\phi_1 = \forall x \exists y (P(x) \rightarrow Q(y)) \rightarrow \forall x (P(x) \rightarrow \forall y Q(y))$
- [8] (b)  $\phi_2 = \forall x \exists y (P(x) \rightarrow Q(y)) \rightarrow \forall x (P(x) \rightarrow \exists y Q(y))$
- [8] (c)  $\phi_3 = \forall x \exists y (P(x) \rightarrow Q(y)) \rightarrow \exists x (P(x) \rightarrow \forall y Q(y))$

4 Consider the model  $\mathcal{M}$ :



- [8] (a) Determine in which states of  $\mathcal{M}$  the CTL formula  $\neg E[\neg p U(\neg p \wedge \neg q)] \wedge \neg EG \neg p$  holds.
- [8] (b) Give an LTL formula  $\phi$  that holds in states 1 and 2 but not in state 0 of  $\mathcal{M}$ .
- [8] (c) Give a model which shows that the CTL\* formulas  $A[FG p]$  and  $A[FA[GP]]$  are not equivalent.