Algorithmische Mathematik 7 — Logic in Computer Science

This exam consists of four exercises. *Explain your answers.* The available points for each item are written in the margin. You need at least 50 points to pass.

1 Consider the following questions concerning propositional logic.

[7] (a) Give a natural deduction proof of the sequent $\neg(p \vee \neg q) \vdash \neg p \wedge q$.

[7] (b) Is the formula $\neg((p \wedge q) \vee (\neg p \wedge r))$ satisfiable?

[7] (c) Test the satisfiability of the formula in (b) with the linear SAT solver.

[7] (d) Test the satisfiability of the formula in (b) with the cubic SAT solver.

2 Consider the boolean function

$$f(x,y,z) = \begin{cases} 0 & \text{if } x \neq y \text{ or } z = \max\{x,y\} \\ 1 & \text{otherwise} \end{cases}$$

[8] (a) Give a binary decision tree for f with the variable ordering $[y,z,x]$ and use the reduce algorithm to construct an equivalent reduced OBDD.

[8] (b) Use Shannon's expansion to obtain a boolean expression that is equivalent to f.

[8] (c) Construct a reduced OBDD for $\forall z.f$ by using apply and restrict. Give all intermediate OBDDs.

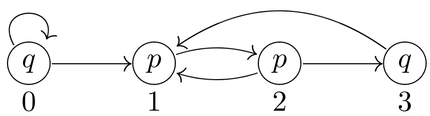
3 For each of the following formulas of predicate logic, either give a natural deduction proof or find a model which does not satisfy it:

[8] (a) $\phi_1 = \exists x\, \forall y\, (P(x) \to Q(y)) \to \forall y\, (\exists x\, P(x) \to Q(y))$

[8] (b) $\phi_2 = \exists x\, \forall y\, (P(x) \to Q(y)) \to \forall y\, (\forall x\, P(x) \to Q(y))$

[8] (c) $\phi_3 = \exists x\, \forall y\, (P(x) \to Q(y)) \to \exists y\, (\forall x\, P(x) \to Q(y))$

4 Consider the model $\mathcal{M}$:



[8] (a) Determine in which states of $\mathcal{M}$ the CTL formula $\neg \mathsf{E}[\neg p\, \mathsf{U} (\neg p \wedge \neg q)] \wedge \neg\, \mathsf{EG}\, \neg p$ holds.

[8] (b) Give an LTL formula ϕ that holds in states 0, 1 and 3 but not in state 2 of $\mathcal{M}$.

[8] (c) Give a model which shows that the CTL* formulas $\mathsf{E}[\mathsf{G}\,\mathsf{F}\, p]$ and $\mathsf{E}[\mathsf{G}\,\mathsf{E}[\mathsf{F}\, p]]$ are not equivalent.