University of Innsbruck — Institute of Computer Science
3rd Exam — 27 April 2007, 9:00–11:00

Algorithmische Mathematik 7 — Logic in Computer Science

This exam consists of four exercises. *Explain your answers.* The available points for each item are written in the margin. You need at least 50 points to pass.

1 Consider the following questions concerning propositional logic.

[7] (a) Give a natural deduction proof of the sequent $\neg\neg p \to q \vdash \neg p \vee q$.

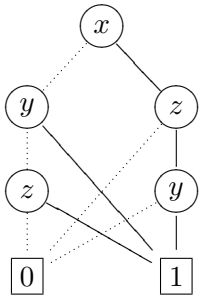
[7] (b) Transform the formula

$$(q \to p) \wedge (s \to t) \wedge ((\neg q \wedge \neg s) \to t) \wedge \neg(t \vee \neg(p \to s))$$

into clausal form.

[7] (c) Use resolution to decide whether the formula in part (b) is satisfiable.

2 Consider the following BDD B:



[7] (a) Is B ordered?

[7] (b) Find an equivalent reduced OBDD with respect to the variable ordering $[y, z, x]$.

[7] (c) Which boolean function does B represent?

[7] (d) How can B be used to determine validity and satisfiability of the formula it represents?

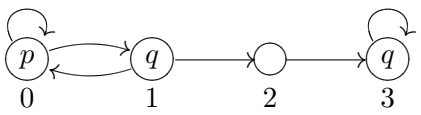
3 For each of the following formulas of predicate logic, either give a natural deduction proof or find a model which does not satisfy it:

[9] (a) $\phi_1 = \forall x\,(\forall y\,P(x, s(y)) \to (x = a)) \to P(a, a)$

[9] (b) $\phi_2 = \forall x\,(P(x) \wedge Q(a)) \to \neg\forall x\,\neg(P(x) \wedge Q(x))$

[9] (c) $\phi_3 = (\exists x\,(P(x) \wedge Q(x)) \wedge \neg\forall x\,Q(x)) \to \forall y\,(Q(y) \to P(y))$

4 Consider the model $\mathcal{M}$:



[8] (a) Determine in which states of $\mathcal{M}$ the CTL formula $\mathsf{AG}\,\mathsf{EF}\,q \to \mathsf{A}[p\,\mathsf{U}\,q]$ holds.

[8] (b) Give an LTL formula ϕ that holds only in states 0 and 1 of $\mathcal{M}$.

[8] (c) Give a model which shows that the CTL* formulas $\mathsf{A}[\mathsf{F}\,\mathsf{A}[\mathsf{X}\,p] \vee \mathsf{G}\,\neg p]$ and $\mathsf{F}\,\mathsf{X}\,p \vee \mathsf{G}\,\neg p$ are not equivalent.