University of Innsbruck | Institute of Computer Science
1st Exam | 31 January 2008, 08:00–11:00

Logik | WS 2007/2008 | LVA 703019

This exam consists of five exercises. The available points for each item are written in the margin. You need at least 50 points to pass.

1 For each of the following sequents, either give a natural deduction proof or explain why such a proof does not exist.

[6] (a) $\vdash (p \to q) \to ((\neg p \to q) \to q)$

[6] (b) $p \to \neg q \vee r, \neg r \vdash \neg q \to \neg p$

[7] (c) $\vdash \neg((p \to q) \wedge ((r \to p) \wedge (((\neg p \wedge \neg r) \to p) \wedge \neg(p \vee \neg(s \to q)))))$

2 Consider the boolean function

$$f(x,y,z) = \begin{cases} 0 & \text{if } x = z \text{ or } x = \min\{y,z\} \\ 1 & \text{otherwise} \end{cases}$$

[6] (a) Give a binary decision tree for f with the variable ordering $[x,y,z]$ and use the reduce algorithm to construct an equivalent reduced OBDD.

[6] (b) Use Shannon's expansion to obtain a boolean expression that is equivalent to f.

[7] (c) Construct a reduced OBDD for $\exists y.f$ by using apply and restrict. Give all intermediate OBDDs.

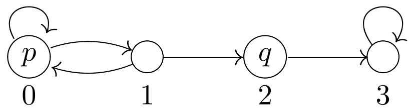
3 For each of the following formulas of predicate logic, either give a natural deduction proof or find a model which does not satisfy it:

[7] (a) $\phi_1 = (\forall x\,(P(x) \to Q(x)) \wedge P(a)) \to (b = a \to Q(b))$

[7] (b) $\phi_2 = \exists x\,\forall y\,(R(x,x) \to R(x,y)) \to (\forall x\,\forall y\,R(x,y) \vee \neg\forall x\,R(x,x))$

[7] (c) $\phi_3 = (\forall x\,R(a,x) \wedge \forall x\,\forall y\,(R(x,y) \to R(f(x),f(y)))) \to \forall x\,R(x,f(x))$

[7] **4** Consider the model $\mathcal{M}$:



[7] (a) Determine in which states of $\mathcal{M}$ the CTL formula $\phi = \mathsf{E}[p\,\mathsf{U}(\neg p \wedge \mathsf{EX}\,q)]$ holds.

[7] (b) Transform ϕ into an equivalent CTL formula ψ without the connective EX.

[7] (c) Give an LTL formula χ that holds only in states 0 and 2 of $\mathcal{M}$.

[20] **5** Determine whether the following statements are true or false. Every correct answer is worth 2 points. For every wrong answer 1 point is subtracted, provided the total number of points is non-negative.

statement
Propositional Horn formulas admit an efficient algorithm for validity checking.
CTL and LTL have the same expressive power.
Tseitin's transformation transforms propositional formulas into equivalent conjunctive normal forms.
$\forall x(\phi \to \psi) \dashv\vdash \exists x\,\phi \to \psi$
For every CTL formula ϕ, $[\![\,\mathsf{AF}\,\phi\,]\!]$ is a least fixed point of the function $X \mapsto [\![\,\phi\,]\!] \cup \mathsf{pre}_\forall(X)$.
Validity in propositional logic is undecidable.
Every adequate set of CTL connectives contains AU.
Every predicate logic formula has an equivalent Skolem normal form.
$\{\oplus, \leftrightarrow, \neg\}$ is an adequate set of propositional connectives.
The LTL formulas $\phi\,\mathsf{U}\,\psi$ and $\neg(\neg\psi\,\mathsf{U}\,(\neg\phi \wedge \neg\psi)) \wedge \mathsf{F}\psi$ are semantically equivalent.