Logik | WS 2007/2008 | LVA 703019

This exam consists of five exercises. The available points for each item are written in the margin. You need at least 50 points to pass.

1 For each of the following sequents, either give a natural deduction proof or explain why such a proof does not exist.

[5] (a) $\vdash (p \to q) \lor (q \to r)$

[6] (b) $\neg r \to p \lor q, r \land \neg q \vdash r \to q$

[6] (c) $p \lor q, \neg q \lor r \vdash p \lor r$

[7] **2** (a) Write a Prolog program to solve the following cryptarithmetic puzzle:

```
      E X A M
P R E P A R E  +
-------------
S U C C E S S
```

You don't have to write the *use_module* statement. Moreover, the puzzle need not be solved.

[7] (b) Write a Prolog program which computes the sum of all prime numbers in a list of numbers. (Hint: `X mod Y` computes X modulo Y.)

[7] (c) Consider the following Prolog program:

```
p(X,a) :- p(X,b), !, p(X,X), p(X,c).
p(f(X),b).
p(X,f(X)).
p(f(a),X).
p(X,c).
```

Write down all SLD derivations starting from the query

```
?- p(X,Y).
```

and give the list of answers in the same order as they will be returned by a Prolog system. You may represent the SLD derivations as a tree to share those parts that are common to more than one SLD derivation.

3 For each of the following formulas of predicate logic, either give a natural deduction proof or find a model which does not satisfy it:

[7] (a) $\phi_1 = \forall x\, (P(x) \lor Q(x)) \to \forall x\, P(x) \lor \forall x\, Q(x)$

[7] (b) $\phi_2 = \forall x\, (P(x) \lor Q(x)) \to \forall x\, P(x) \lor \exists x\, Q(x)$

[7] (c) $\phi_3 = \forall x\, (P(x) \lor Q(x)) \to (\exists x\, \neg Q(x) \to (\forall x\, (R(x) \to \neg P(x)) \to \exists x\, \neg R(x)))$

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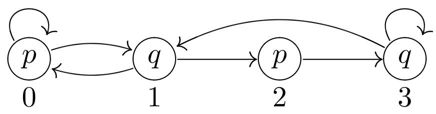
4 Consider the model $\mathcal{M}$:



[7] (a) Determine in which states of $\mathcal{M}$ the CTL formula $\phi = \mathsf{A}[p \,\mathsf{U}\, q] \to \mathsf{E}[(p \to \mathsf{AX}\, q) \,\mathsf{U}\, \mathsf{EG}\, q]$ holds.

[7] (b) Give an LTL formula ψ that holds only in states 0 and 3 of $\mathcal{M}$.

[7] (c) Give a model which shows that the CTL* formulas $\mathsf{A}[(\mathsf{X}\, p \vee \mathsf{X}\,\mathsf{X}\, p)]$ and $\mathsf{AX}\, p \vee \mathsf{AX}\,\mathsf{AX}\, p$ are not equivalent.

[20] 5 Determine whether the following statements are true or false. Every correct answer is worth 2 points. For every wrong answer 1 point is subtracted, provided the total number of points is non-negative.

statement
Prolog programs can be executed using natural deduction for propositional logic.
$\{\oplus, \to\}$ is an adequate set of propositional connectives.
CTL* is more expressive than LTL.
Every monotone function $F : \mathcal{P}(\mathbb{N}) \to \mathcal{P}(\mathbb{N})$ has a fixed-point.
$\{\bot, \neg, \wedge, \mathsf{X}, \mathsf{W}\}$ is an adequate set of connectives for LTL.
Satisfiability in predicate logic is decidable.
A sentence ψ in predicate logic has a model with infinitely many elements if for all $n \geqslant 1$ ψ has a model with at most n elements.
The boolean function $f(x, y) = (x \oplus y) + y$ is self-dual.
Every predicate logic formula has an equivalent prenex normal form.
Every boolean function has a unique reduced OBDD.