UNIVERSITY OF INNSBRUCK — INSTITUTE OF COMPUTER SCIENCE

3RD EXAM — 22 SEPTEMBER 2008, 09:00–12:00

Logik — WS 2007/2008 — LVA 703019

This exam consists of <u>five</u> exercises. The available points for each item are written in the margin. You need at least 50 points to pass.

1 For each of the following sequents, either give a natural deduction proof or explain why such a proof does not exist.

[5] (a) $p \vee q \vdash \neg(\neg p \vee q)$

[6] (b) $\neg(\neg p \vee \neg q) \vdash (p \wedge q)$

[6] (c) $\vdash ((p \to q) \to q) \to ((q \to p) \to p)$

2 Consider the boolean function

$$f(x,y,z) = \begin{cases} 0 & \text{if } x = z \text{ or } y = \min\{x,z\} \\ 1 & \text{otherwise} \end{cases}$$

[7] (a) Give a binary decision tree for f with the variable ordering $[z,y,x]$ and use the reduce algorithm to construct an equivalent reduced OBDD.

[7] (b) Use Shannon's expansion to obtain a boolean expression that is equivalent to f.

[7] (c) Construct a reduced OBDD for $\forall z.f$ by using apply and restrict. Give all intermediate OBDDs.

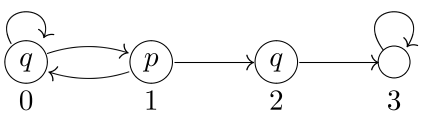
3 For each of the following formulas of predicate logic, either give a natural deduction proof or find a model which does not satisfy it:

[7] (a) $\phi_1 = \exists x\,(P(x) \vee Q(x)), \forall x\,\neg Q(x), \forall x\,(R(x) \to \neg P(x)) \vdash \exists x\,\neg R(x)$

[7] (b) $\phi_2 = \exists x\,(P(x) \vee Q(x)), \exists x\,\neg Q(x), \forall x\,(R(x) \to \neg P(x)) \vdash \exists x\,\neg R(x)$

[7] (c) $\phi_3 = \forall x\,(P(x) \vee Q(x)), \exists x\,\neg Q(x), \forall x\,(R(x) \to \neg P(x)) \vdash \exists x\,\neg R(x)$

4 Consider the model $\mathcal{M}$:



[7] (a) Determine in which states of $\mathcal{M}$ the CTL formula $\phi = \mathsf{A}[p\,\mathsf{U}\,q] \to \mathsf{E}[(p \to \mathsf{AX}\,q)\,\mathsf{U}\,\mathsf{EG}\,q]$ holds.

[7] (b) Transform ϕ into an equivalent CTL formula ψ without the connective EG.

[7] (c) Give an LTL formula χ that holds only in states 1 and 2 of $\mathcal{M}$.

Turn Over **Turn Over** **Turn Over**

[20] 5 Determine whether the following statements are true or false. Every correct answer is worth 2 points. For every wrong answer 1 point is subtracted, provided the total number of points is non-negative.

statement
LTL is more expressive than CTL.
Every boolean function admits a minimum size reduced BDD which is ordered.
Every predicate logic formula has an equisatisfiable Skolem normal form.
Every function $F : \mathcal{P}(A) \to \mathcal{P}(A)$ with finite A has a greatest fixed-point.
A logic program is a finite non-empty set of clauses.
$\{\oplus, \wedge, \vee\}$ is an adequate set of propositional connectives.
$\forall x\, \phi \wedge \forall x\, \psi \dashv\vdash \neg\exists x\, (\neg\phi \vee \neg\psi)$
A set Γ of sentences of predicate logic is unsatisfiable if and only if all finite subsets of Γ are unsatisfiable.
The natural deduction proof rules LEM, PBC and $\neg\neg$i are inter-derivable with respect to the basic proof rules.
$\{\neg, \vee, \mathsf{AX}, \mathsf{EF}, \mathsf{EU}\}$ is an adequate set of connectives for CTL.