EXAM 1 January 22, 2009

This exam consists of five exercises. The available points for each item are written in the margin. You need at least 50 points to pass.

1 Consider the boolean function $f(x, y, z) = x \oplus xy \oplus yz$.

[9] (a) Give a binary decision tree for f with the variable ordering $[x, y, z]$ and use the reduce algorithm to construct an equivalent reduced OBDD.

[9] (b) Determine all minimal adequate subsets of $\{\to, \vee, \bot, f\}$.

[8] **2** (a) Transform the formulas

i. $\forall x\, \neg \forall y\, (\forall z\, P(x, z) \to Q(f(y), x)) \vee \exists x\, \forall y\, P(x, y)$

ii. $\exists x\, ((\forall y\, P(x, y) \to \forall y\, (P(x, y) \wedge Q(y, x))) \to \forall y\, Q(x, y))$

into equisatisfiable clausal forms.

[12] (b) Use resolution to determine whether the clausal forms

i. $\{\, \{P(x, y), Q(x, f(y)), R(x, y)\},$
$\{\neg P(x, b), Q(x, f(y)), \neg R(x, f(y))\},$
$\{\neg R(x, b)\},$
$\{R(f(a), f(x))\},$
$\{\neg Q(f(a), f(b))\}\,\}$

ii. $\{\, \{\neg P(x, y), Q(b, x), R(x, x)\},$
$\{P(x, y), \neg Q(z, f(u)), R(f(a), z)\},$
$\{P(x, y), Q(f(a), y), R(z, b)\},$
$\{\neg R(x, x)\},$
$\{\neg Q(b, f(x))\}\,\}$

are satisfiable. (Here a, b are constants and u, x, y, z variables.)

3 For each of the following formulas of predicate logic, either give a natural deduction proof or find a model which does not satisfy it:

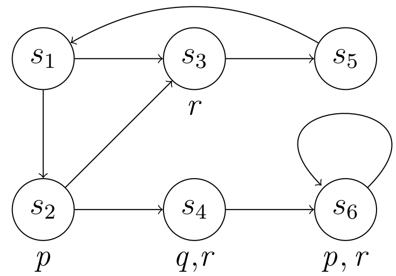
[7] (a) $\phi_1 = \forall x\, \exists y\, (P(y) \to Q(x)) \to \forall x\, (\exists y\, P(y) \to Q(x))$

[7] (b) $\phi_2 = \forall x\, \exists y\, (P(x) \to Q(y)) \to \forall x\, (P(x) \to \exists y\, Q(y))$

[7] (c) $\phi_3 = \forall x\, (\exists y\, P(y) \to Q(x)) \to \exists y\, \forall x\, (P(y) \to Q(x))$

Turn Over **Turn Over** **Turn Over**

4 Consider the CTL formulas $\phi = \mathsf{E}[p \vee \neg r \,\mathsf{U}\, \mathsf{EX}\,\mathsf{AG}\,\neg q]$ and $\psi = \mathsf{AG}\,\mathsf{AF}(p \vee q)$, the LTL formula $\chi = \mathsf{GF}p \vee \mathsf{GF}q$, and the model $\mathcal{M}$:



[7] (a) Apply the CTL model checking algorithm to determine the states of $\mathcal{M}$ which satisfy ϕ.

[7] (b) Provide an LTL formula ξ which does not contain the proposition r such that $\mathcal{M}, s \models \xi$ if and only if $s = s_3$.

[7] (c) Determine whether ψ is equivalent to χ. In the negative case also provide a counterexample.

[20] 5 Determine whether the following statements are true or false. Every correct answer is worth 2 points. For every wrong answer 1 point is subtracted, provided the total number of points is non-negative.

statement
Every clause with the property that two of its literals unify has a factor.
To represent the transition relation of a CTL model with 16 states we need at most 8 boolean variables.
$\forall x\,(\exists x\,\phi \rightarrow \psi) \dashv\vdash \exists x\,\phi \rightarrow \forall x\,\psi$
Executing the Prolog query `?- select(b,[a,b,c,d],X).` produces the answer `X = [a,c,d]`.
A set of sentences of predicate logic is satisfiable if and only if all finite subsets are satisfiable.
A boolean function $f\colon \{0,1\}^n \rightarrow \{0,1\}$ is self-dual if $f(x_1,\dots,x_n) = f(\overline{x_1},\dots,\overline{x_n})$.
Every CTL* path formula is a CTL* state formula.
If the set S has n elements then $F^n(S)$ is the least fixed point of a monotone function $F\colon \mathcal{P}(S) \rightarrow \mathcal{P}(S)$.
Every Horn formula is satisfiable.
The LTL formulas $\neg(\phi \,\mathsf{R}\, \psi)$ and $\neg\psi \,\mathsf{U}\, \neg\phi$ are semantically equivalent.