institut für informatik

Logik | WS 2008/2009 | LVA 703019

EXAM 2 | April 17, 2009

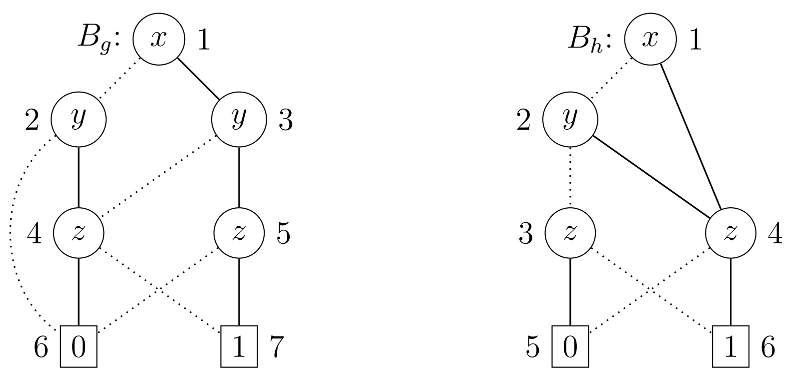
This exam consists of five exercises. The available points for each item are written in the margin. You need at least 50 points to pass.

[6] **1** (a) Give a natural deduction proof of the sequent $\vdash ((p \to q) \to p) \to p$

[6] (b) Test the satisfiability of the formula $\neg(\neg p \wedge \neg(p \wedge q)) \wedge \neg(\neg(p \wedge q) \wedge \neg q)$ with the linear SAT solver.

[6] (c) Test the satisfiability of the formula in (b) with the cubic SAT solver.

2 Consider the two OBBDs



[7] (a) Compute $\mathsf{apply}(\oplus, B_g, B_h)$.

[6] (b) Use Shannon's expansion to obtain boolean expressions that are equivalent to B_g and B_h.

[7] (c) Use $\mathsf{restrict}$ to compute a reduced OBDD for $g[1/x][0/z]$, where g is the boolean formula represented by B_g. Give all intermediate OBDDs.

3 For each of the following sequents of predicate logic, either give a natural deduction proof or find a model which does not satisfy it:

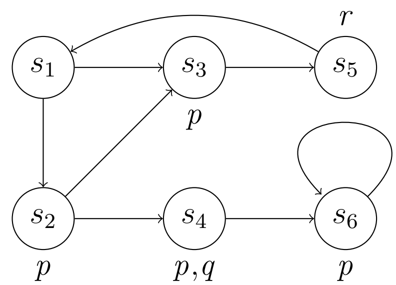
[7] (a) $\forall x\, \forall y\, \forall z\, (R(x, y) \wedge R(y, z) \to R(x, z)), \forall x\, \neg R(x, x) \vdash \forall x\, \forall y\, (R(x, y) \wedge R(y, x) \to x = y)$

[7] (b) $\forall x\, \neg R(x, x), \forall x\, \exists y\, R(x, y) \vdash \neg \exists x\, \forall y\, x = y$

[7] (c) $\exists x\, \forall y\, (\neg R(x, y) \vee \neg S(x, y)) \vdash \forall x\, \exists y\, (R(x, y) \vee S(x, y))$

Turn Over **Turn Over** **Turn Over**

4 Consider the CTL formulas $\phi = \mathsf{EG}(p \rightarrow \mathsf{A}[p\,\mathsf{U}\,r] \vee \mathsf{EF}\,q)$ and $\psi = \mathsf{AG}(\mathit{request} \rightarrow \mathsf{AF}\,\mathsf{AX}\,\mathit{response})$, the LTL formula $\chi = \mathsf{G}(\mathit{request} \rightarrow \mathsf{F}\,\mathsf{X}\,\mathit{response})$, and the model $\mathcal{M}$:



[7] (a) Apply the CTL model checking algorithm to determine the states of $\mathcal{M}$ which satisfy ϕ.

[7] (b) Provide a model and a state which distinguishes ψ from χ. Indicate which of the two formulas is satisfied.

[7] (c) Provide a CTL formula ξ such that $\xi \equiv \chi$.

[20] 5 Determine whether the following statements are true or false. Every correct answer is worth 2 points. For every wrong answer 1 point is subtracted, provided the total number of points is non-negative.

statement
Executing the Prolog query `?- 1+2 = 3.` produces the answer `Yes`.
$\forall x\,(\phi \rightarrow \exists x\,\psi) \dashv\vdash \exists x\,\phi \rightarrow \exists x\,\psi$
The CTL formulas $\mathsf{AF}\,\neg\phi$ and $\mathsf{A}[\bot\,\mathsf{U}\,\neg\phi]$ are semantically equivalent.
A propositional formula ϕ is valid if and only if $\neg\phi$ is satisfiable.
The boolean function $f(x, y) = x + y$ is affine.
Every adequate set of LTL connectives contains U.
The rule $\neg\neg$e is derivable from the basic proof rules of natural deduction.
$\{\|\}$ is an adequate set of propositional connectives, where $x \,\|\, y = \neg(x \vee y)$.
For all CTL formulas ϕ and ψ, $[\![\,\mathsf{E}[\phi\,\mathsf{U}\,\psi]\,]\!]$ is a least fixed point of the function $X \mapsto [\![\,\psi\,]\!] \cap ([\![\,\phi\,]\!] \cup \mathsf{pre}_{\exists}(X))$.