EXAM 3 September 25, 2009

This exam consists of five exercises. The available points for each item are written in the margin. You need at least 50 points to pass.

**1** Consider the boolean function $f(x, y, z) = 1 \oplus x \oplus y \oplus z$.

[9] (a) Give a binary decision tree for $f$ with the variable ordering $[x, y, z]$ and use the reduce algorithm to construct an equivalent reduced OBDD.

[9] (b) Determine all minimal adequate subsets of $\{\rightarrow, \wedge, \top, f\}$.

[4] **2** (a) Write a Prolog program `suffix/2` which tests whether the list given as second argument is a suffix of the list given as first argument.

[8] (b) Write a Prolog program `sublist/2` which tests whether the list given as second argument is a sublist of the list given as first argument. Examples:

```
?- sublist([1,2,3,4],[2,3])        ?- sublist([1,2,3,4],[2,4])
Yes                                No
```

[8] (c) Consider the following Prolog program:

```
p(X,Y) :- q(X,Z), q(Z,Y).
p(X,b) :- q(b,X), !, p(X,X).
q(a,b).
q(b,f(X)).
q(b,c).
p(X,f(X)).
p(c,c).
```

Write down all SLD derivations starting from the query

```
?- p(X,Y).
```

and give the list of answers in the same order as they will be returned by a Prolog system. You may represent the SLD derivations as a tree to share those parts that are common to more than one SLD derivation.

**3** For each of the following formulas of predicate logic, either give a natural deduction proof or find a model which does not satisfy it:

[7] (a) $\phi_1 = \forall x\, \exists y\, (P(x) \wedge Q(y)) \rightarrow \exists y\, \forall x\, (P(x) \wedge Q(y))$

[7] (b) $\phi_2 = \forall x\, (P(x) \vee Q(x)) \wedge \forall x\, (Q(x) \vee R(x)) \rightarrow \exists x\, (P(x) \vee R(x))$

[7] (c) $\phi_3 = \exists x\, \forall y\, P(x, y) \wedge \exists x\, \forall y\, \neg P(x, y) \rightarrow \forall x\, \exists y\, \forall z\, (P(x, y) \leftrightarrow P(x, z))$

[4] Formulate the statements in parts (4a) and (4b) in predicate logic or in second-order logic if the former is not possible. Here, $R$ and $S$ are unary predicate symbols whereas $P$ and $Q$ are binary predicate symbols.

[7] (a) If $R$ contains at least three elements then $S$ contains at most two elements.

[7] (b) There is a relation in between $P$ and $Q$ (i.e., larger than $P$ and smaller than $Q$) which is transitive.

[7] (c) Give a translation $\tau$ from formulas of propositional logic to formulas of second-order logic such that the the following three statements are equivalent:

- $\phi$ is satisfiable
- $\tau(\phi)$ is satisfiable
- $\tau(\phi)$ is valid

Your translation should be purely syntactical, i.e., defining

$$\tau(\phi) = \begin{cases} \top & \text{if } \phi \text{ is satisfiable} \\ \bot & \text{otherwise} \end{cases}$$

or using a similar definition is not allowed.

[20] [5] Determine whether the following statements are true or false. Every correct answer is worth 2 points. For every wrong answer 1 point is subtracted, provided the total number of points is non-negative.

| statement |
|---|
| $\{\bot, \neg, \wedge, \mathsf{X}, \mathsf{F}, \mathsf{R}\}$ is an adequate set of connectives for LTL. |
| Tseitin's transformation transforms every CNF into the same CNF. |
| $\forall x(\phi \rightarrow \forall y\, \psi) \dashv\vdash \exists x\, \phi \rightarrow \forall y\, \psi$ |
| The unification problem $\{x \stackrel{?}{=} f(y), y \stackrel{?}{=} g(z), z \stackrel{?}{=} x\}$ is in solved form. |
| Executing the Prolog query `?- X = f(X).` produces the answer `No`. |
| $\{\bot, \rightarrow, \leftrightarrow\}$ is an adequate set of propositional connectives. |
| Every ordered BDD is reduced. |
| There is an efficient procedure to test the semantic equivalence of two propositional formulas. |
| The instance $\{(01, 0), (0, 1), (1, 01)\}$ of Post correspondence problem has a solution. |
| An LTL formula $\phi$ is satisfied in a state $s$ of a model $\mathcal{M}$ if there is a path starting in $s$ that satisfies $\phi$. |