Logik	WS 2009/2010	LVA 703019

EXAM 1	February 4, 2010

This exam consists of five exercises. The available points for each item are written in the margin. You need at least 50 points to pass.

1 Consider the boolean function $f(x, y, z) = 1 \oplus xy \oplus yz$.

[10] (a) Give a binary decision tree for f with the variable ordering $[x, y, z]$ and use the reduce algorithm to construct an equivalent reduced OBDD.

[10] (b) Determine all minimal adequate subsets of $\{\rightarrow, \oplus, \bot, f\}$.

2 In this exercise we consider propositional formulas (without implications) which are represented by Prolog terms as follows. The operations $\top$, $\neg$, $\wedge$, and $\vee$ are represented by `true/0`, `not/1`, `and/2`, and `or/2`. Propositional variables are represented by atoms. For example, the formula $x \wedge (\top \vee \neg y)$ is represented by the term `and(x,or(true,not(y)))`.

Note that all parts but (c) can be done independently. For example, in part (b) you can assume that `elem/2` exists, regardless of whether you solved part (a) or not.

You may use the predefined predicate `atom/1` which checks whether the input is an atom.

[2] (a) Write a Prolog predicate `elem/2` where `elem(X,Xs)` should be true if and only if `X` is an element of the list `Xs`.

[6] (b) Write a Prolog predicate `eval/2` where `eval(F,A)` should be true if and only if the formula `F` evaluates to $\top$ under the variable assignment `A`. Here, a variable assignment is represented by the list of those propositional variables (atoms) which evaluate to $\top$.

For example, `eval(and(x,or(true,not(y))),[x,y])` should be provable.

[4] (c) Consider the Prolog program

```
sat(F) :- eval(F,A).
```

to determine satisfiability of a formula. Compute the SLD-tree of `sat(not(x))` for your implementation of `elem/1` and `eval/2`. What does this tree tell you about the completeness of `sat/1`?

[8] (d) Write a Prolog program `nnf/2` which transforms a given formula into negation normal form.

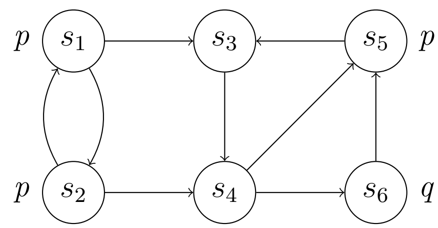
3 For each of the following sequents of predicate logic, either give a natural deduction proof or find a model which does not satisfy it:

[8] (a) $\forall x\, \forall y\, \forall z\, (R(x,y) \wedge R(y,z) \rightarrow R(x,z)), \forall x\, \neg R(x,x) \vdash \neg \exists x\, \exists y\, (R(x,y) \wedge R(y,x))$

[6] (b) $\forall x\, \exists y\, P(x,y), \forall x\, \forall y\, (P(x,y) \rightarrow Q(x,y)) \vdash \exists x\, \forall y\, Q(x,y)$

[6] (c) $\exists x\, \forall y\, P(x,y), \forall x\, \forall y\, (P(x,y) \rightarrow Q(x,y)) \vdash \forall y\, \exists x\, Q(x,y)$

4 Consider the CTL formula $\phi = \mathsf{A}[p\, \mathsf{U}\, \neg p \wedge \mathsf{AX}\, \mathsf{EG}\, \neg q]$, and the model $\mathcal{M}$:



[10] (a) Apply the CTL model checking algorithm to determine the states of $\mathcal{M}$ which satisfy ϕ.

[5] (b) Provide a CTL formula ψ which is equivalent to ϕ such that ψ does not contain the CTL connectives AU, AX and EG.

[5] (c) Provide a CTL formula χ that is satisfied only in state s_1 of $\mathcal{M}$.

[20] **5** Determine whether the following statements are true or false. Every correct answer is worth 2 points. For every wrong answer 1 point is subtracted, provided the total number of points is non-negative.

statement
Every adequate set of temporal CTL connectives contains AU.
The propositional formulas $\phi \leftrightarrow \psi \vee \chi$ and $(\psi \vee \chi \vee \neg\phi) \wedge (\neg\psi \vee \phi) \wedge (\neg\chi \vee \phi)$ are equivalent.
A unary boolean function f is monotone if $\overline{f(0)} + f(1) = 1$.
Executing the Prolog query `?- X =:= 2+3.` produces the answer `X = 5`.
Satisfiability in predicate logic is undecidable.
For any set C of fairness constraints, $\mathsf{E}_C\mathsf{X}\,\phi \;\equiv\; \mathsf{EX}(\phi \wedge \mathsf{E}_C\mathsf{G}\,\top)$.
$\forall x\, \phi \vee \forall x\, \psi \dashv\vdash \forall x\, (\phi \vee \psi)$
The algebraic normal form of the boolean function $f(x,y) = x + \overline{y}$ is $1 \oplus x \oplus xy$.
The term $f(x,y)$ is free for z in $\forall x\, ((\forall z\, (P(z) \wedge Q(y))) \rightarrow \neg P(x) \vee Q(z))$.
$[\![\, \mathsf{E}[\phi\, \mathsf{U}\, \psi]\,]\!]$ is the least fixed point of the function $F\colon \mathcal{P}(S) \rightarrow \mathcal{P}(S)$ that maps X to $[\![\, \phi\,]\!] \cup ([\![\, \psi\,]\!] \cap \mathsf{pre}_{\exists}(X))$.